Acoustical Equalization at Multiple Listening Positions

by

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Acoustical Equalization at Multiple Listening Positions

submitted by Bruno Korst-Fagundes

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Abstract

This thesis presents an entirely new approach for the acoustical equalization of a room at multiple listening positions. The technique makes use of a mathematical tool called the condition number to systematically identify and deal with frequencies that impose difficulties to the inversion of a matrix of room transfer functions. For problematic frequencies, a new inverse of the transfer function matrix is calculated to be used in conjunction with a set of desired gains, in order to achieve a very close approximation of the direct inverse of such matrix. This thesis presents results from data collected in a car, and proposes a real-time implementation of a room equalizer.

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Thanks are to be given:

To God: Creator, Almighty and Most Merciful. Triune after and before all.

To my family, of blood and faith. Simultaneously far away and next door, both their support and encouragement were most precious to me at all times.

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"É o ixforrrrço final..."

by *Dona Irene*, since 1978

(*Carioca* accent added for her benefit)

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CHAPTER 1

Introduction and Thesis Outline

Motivation

Historically, providing listeners with a clear sound coming from the presenter has always been one of the greatest challenges and concerns for architects and sound system designers. From the sound system designer's point of view, the first concern is to find an optimal position for the loudspeakers within a venue.

A second concern is to correct the sound that is sent through the loudspeakers in order to provide the listeners with a faithful reproduction of the signal being sent to those loudspeakers. This correction is done in order to achieve the best distribution of sound over a certain area or at particular positions within a venue. Each case, however, would require different correction schemes, now called equalization.

The simultaneous equalization of the overall transfer function at several listening positions is a very hard task to perform. This is due to the variability of the transfer function of a room from position to position. In that case, the equalizer has to deal with many transfer functions and possibly make use of several sound sources to achieve satisfactory results.

The search for an algorithm that allows for the simultaneous equalization of a certain audible frequency band for various listener positions in a venue is the motivation for this thesis, and its objective is to present a novel approach to achieve acoustical equalization of a room in multiple listening positions.

Introduction

Within the past 10 years, emphasis has been placed on algorithms to perform equalization, making use of Digital Signal Processing techniques [Elliott89] [Munshi90] [Genrx93] [Mourj94] [Korst95]. Equalizing sound coming from a loudspeaker to a particular listening position means pre-filtering the sound with a filter of transfer function equal to the inverse transfer function of the room from sound source to listening position.

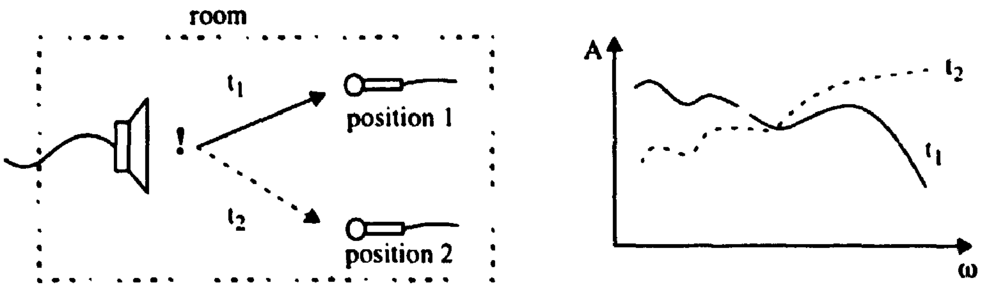
This is a particularly hard task to perform because:

a) Room impulse responses are very long and require very high order filters (large number of taps) to characterize that response. Very long filters immediately imply heavy computation.

b) Room transfer functions vary for different positions. Equalizing the response for one position will change (in fact, degenerate [Mourj85]) the response in other positions. This problem can also be viewed in two other ways:

> The transfer function of a room from a particular loudspeaker position to a particular listener position may present deep notches for some frequencies, which means that a high-Q peak will appear in the transfer function of the inverse filter. An inverse filter with a peak in its transfer function would then necessarily cause an objectionable peak at a location other than the one being equalized.
>
> The use of only one sound source will not allow for the equalization of more than one position, since it could be that the inverse filter is required to

perform "contradictory" tasks. This is, in fact, verified in [Elliott94]. Schroeder [Schr54] was the first to point out that each listening position is subjected to an unique pattern of reflections, making the transfer functions distinct and often vastly dissimilar. That again indicates that if such an equalization scheme is applied, one listener would receive the equalized signal at a position while another listener would receive a distorted signal somewhere else in the venue. Figure 1 illustrates the problem of equalizing two positions with only one sound source.



transfer functions to be simultaneously equalized

FIGURE 1. Attempt to equalize two positions with one loudspeaker

For the case illustrated in figure 1, the correction needed for t_1 goes against the one needed for t_2. If only one sound source can be used, correcting and improving one listener's sound will impair even further that of the other.

c) Room impulse responses measured from a sound source to a listening position have pure delay, indicating that the inverse filter would be non-causal.

Many approaches have been presented for Single-point Equalization [Genrx93] and for Multipoint Equalization [Munshi90][Elliott94].

Single point equalization is particularly important in studio control rooms, where the response of the room needs to be equalized for a single position of the listener, that is, the recording engineer.

A similar approach can be taken for equalization at multiple points. However, problems other than those listed before arise, when many points are to be equalized. The novel approach for multipoint room equalization to be presented in this thesis addresses these problems in detail, making use of well known mathematical tools such as the condition number, and through generalization of concepts such as the inversion of nearly singular matrices. This new approach will be analyzed from its simplest configuration, a system with two loudspeakers and two listening positions (2x2), to a hypothetical system with n speakers and n listening positions (nxn).

Experimental results for different attempts of 2x2 systems will be presented to confirm the feasibility of the algorithm described. Finally, a real-time implementation of an equalizer will be proposed.

Contributions

This thesis addresses the problems previously discussed and deals with them in order to achieve a feasible solution for the equalization problem. Contributions are made in the following aspects:

1. A novel solution is found for the problem of equalizing the transfer function of a venue at multiple listening positions.

Instead of searching for inverses of each transfer function from sound source to listening position or for a compromise solution such as [Elliott94], this thesis shows how to organize these transfer functions in a matrix and look for the inverse of that matrix.

This is done through an algorithm that systematically identifies and fixes the problems at frequencies where the matrix of transfer functions is nearly singular. A new inverse for the transfer function matrix is then devised to be applied with a desired set of gains, to the signal sent to the sound sources. Both the new inverse and the set of gains will emulate the direct inverse of the matrix of transfer functions, and achieve the acoustical equalization at multiple listening positions.

2. An implementation of a real-time equalizer is proposed, to run on a DSP-based platform.

The proposed system makes use of multirate signal processing techniques and frequency domain filtering (FFT-based FIR). By using multirate techniques one allows for more time to perform the filtering, which is done in the frequency domain, since it is particularly suitable for filters with long impulse responses.

Thesis Outline

This thesis is divided into five chapters and two appendices, which are developed as follows:

This chapter (Chapter 1) presented an introduction to the equalization problem and the motivation for the research developed in multipoint room equalization. The main contributions of this thesis to the research of acoustical equalization of multiple positions in a venue were also presented.

The theoretical background is presented in Chapter 2. Concepts such as impulse response, frequency response of a room and inverse filtering are presented, together with a review of matrix algebra. That chapter also presents a review of previous work done in psychoacoustics and room equalization, with an analysis of the most relevant publications.

Chapter 3 is the main chapter of this thesis. That chapter presents the novel algorithm for multipoint room equalization. The identification of problematic frequencies, calculation of a new inverse for the matrix of transfer functions and the calculation of a set of desired gains are the main features of the algorithm presented. A spatial interpretation of the solution for the multipoint equalization algorithm illustrates the novel approach. Numerical examples are presented for every step of the algorithm.

Chapter 4 presents results of the multipoint equalization algorithm applied to sets of experimental data collected in a car. A system configuration of two sound sources used for the equalization of two listening positions is used. Two attempts are shown to demonstrate the functionality of the novel algorithm. The results obtained are discussed and compared.

In Chapter 5, a real-time implementation for the multipoint equalizer is proposed. The system proposed features multirate signal processing and FIR filtering in the frequency domain (FFT based) as its main techniques for an implementation of the equalizer. At the time this thesis was being prepared, the system lacked a last connection between the decimation and interpolation stages and the main frequency domain filter. This chapter describes the design intent of the system.

All sets of experimental data and results are presented in the two appendices. In Appendix A, measurements of impulse responses and frequency responses for all possibilities of systems with two sound sources used in the equalization of two listening positions are presented. Condition number plots are also presented for 3x3 and 4x4 systems. Appendix B contains all the results of equalization obtained for combinations of two sound sources and two listening positions. The results are briefly discussed and compared. Finally, Appendix C presents key routines used in the simulation and real-time implementation.

CHAPTER 2

Theoretical Background and Previous Work

Introduction

This chapter is divided into two main parts. The first part will present the general theoretical background of room acoustics, where concepts of physical acoustics and psychoacoustics will be described. Still within the first part, a review of basic principles of control theory and linear algebra as applied to the room equalization problem will be presented. The second portion of the chapter will present previous research done in the fields of room acoustics and room equalization.

Room Acoustics - Physical Aspects

The physical side of room acoustics involves the use of well known techniques from linear systems and control theory, such as the impulse response, in order to draw conclusions about the physical characteristics of a venue.

The impulse response of a room is measured by applying an impulsive excitation to the room through a sound source, and collecting the data through a microphone at a certain listening position. The signal collected is the convolution of the impulsive excitation with the impulse response of the room.

Mathematical Formulation

This simple sounding problem can be re-written in more mathematical terms as follows: considering a room a linear time-invariant system [Mourj85], this system can be represented by a response $h(t)$ to an impulse-like excitation applied at $t=0$. Then, the output $v_o(t)$ for an input $v_i(t)$ is given by the convolution of $v_i(t)$ and $h(t)$, as in equation 1.

$$v_o(t) = v_i(t) \otimes h(t) \qquad \text{(EQ 1)}$$

Impulse response and frequency response are related to one another through the Fourier Transformation [Papo77]. A similar formulation, therefore, can be derived from the Fourier transforms of each element of Equation 1. Convolution in the time domain corresponds to multiplication in the frequency domain [Oppenh89]. Equation 2 is the frequency domain representation of Equation 1.

$$V_o(\omega) = H(\omega) \cdot V_i(\omega) \qquad \text{(EQ 2)}$$

The response of the room is now regarded as the frequency response or complex transfer function of the room, $H(\omega)$.

It is beyond the scope of this thesis to derive the formulation related to the convolution theorem [Oppenh89] and Fourier transformation [Papo62], since it can be found in any basic control systems theory literature. For the study developed in this thesis, it suffices to know the basic relationship between impulse and frequency response, extensively applied in room acoustics:

$$h(t) \overset{\mathcal{F}}{\leftrightarrow} H(\omega) \qquad \text{(EQ 3)}$$

From this formulation, one can conclude that the impulse response of a system (time domain) and its transfer function (frequency domain) are related through the Fourier

Transform (for proof, see [Papo77] page 58). By measuring the impulse response, one is therefore capable of obtaining the frequency response (transfer function) of a system.

Similarly for discrete-time systems:

$$h[n] \overset{\mathcal{F}}{\leftrightarrow} H(\omega) \tag{EQ 4}$$

The transformation used is now the Discrete Fourier Transform (DFT), and $H(\omega)$ is the frequency response of the linear time-invariant discrete-time system whose impulse response is $h[n]$.

Aspects of the impulse response of a venue shall now be presented, and some important features extracted from that measurement described.

Impulse Response - what does it tell us?

In room acoustics, the impulse response is the most basic measurement needed by the acoustician to analyze room behaviour. Features such as delay time, early and late reflections and reverberation time are observed by extracting that response. After measuring the impulse response, further calculation using the Fourier transformation of the data provides the frequency response. This response holds information on magnitude and phase over frequency. Figure 2 shows an example of an impulse response, extracted in a car.

The impulse response can be measured in many different ways. Early measurement systems attempted to generate and input to a venue a large amount of energy in a very short period of time (or a "peak") [B&K78]. The measurement of the response at any particular location would then indicate the delay between source and listening position, the early and late reflections, and other characteristics. Modern systems use signals such as maximum-length sequences (MLS) to extract the response [Schr79]. The system used

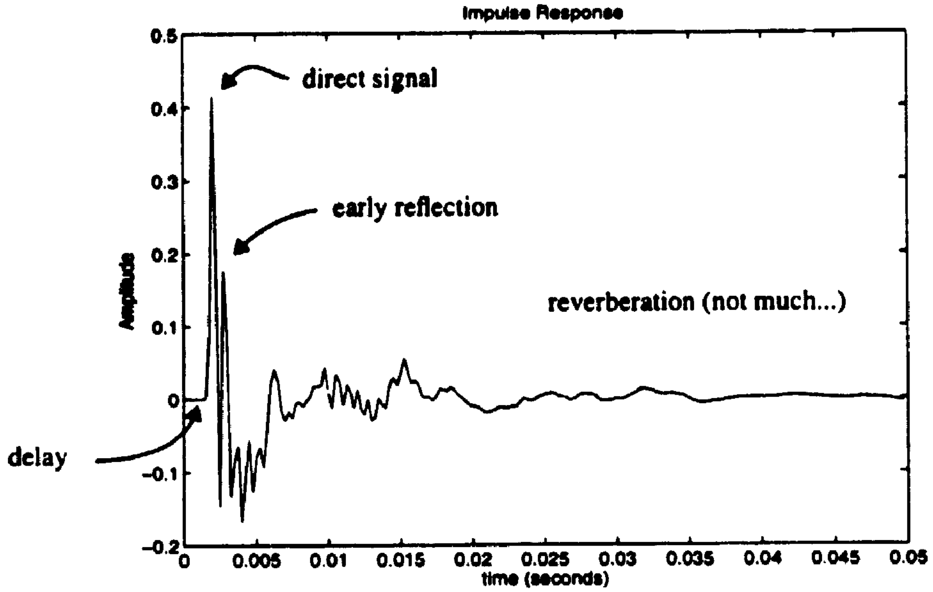


FIGURE 2. Impulse Response of a venue

in this thesis for the collection of data is MLS-based and will be described in more detail in chapter 5. This system - called MLSSA - is commercially available and fairly popular among acousticians due to its accuracy, portability and ease of use.

From the impulse response, one can extract other physical characteristics of the venue. Some of these are listed below.

a) Delay time - This is shown as a flat delay in the impulse response plot. It indicates the time taken by the signal to travel directly from the sound source to the position where the measurement was taken (the listening position). The earliest signal to arrive from different directions at a listening position dominates the perceived localization. This is known as the Haas effect [Olive89].

b) Early and late reflections - Early reflections are represented by the initial portion of the impulse response plot after the direct signal. They are usually clustered and close to the peak relative to the direct signal and within 40ms [Bena85] from the direct signal in small reverberant rooms. Late reflections are noticed towards the "tail" of the impulse response, and sometimes also appear in smaller clusters. For the impulse response presented in figure 2, there is not an easy distinction in terms of time of arrival. The reflections drop in amplitude by a considerable amount after 30ms.

c) Reverberation time - Reverberation time is a standard measurement in room acoustics. Reverberation can be understood as the reflected sound becoming too "dense", so the listener cannot discretely recognize the original sound. This is heard as a tight collection of echoes travelling in all directions, with intensity somewhat independent of the position of the listener in the room[Gard92].

If the impulse response of a room is plotted in terms of decibels, the reverberation of rooms is observed to decay exponentially in time. Reverberation time is defined as the time at which the reflected signal decays by 60dB from the direct signal. This measurement is also known as "RT60" or "T60". Very good descriptions of reverberation can be found in [Gard92], [Gries89] and [Bera54].

As has been formuiated before, one can extract the Fourier Transform (in this case a Fast Fourier Transform) of the impulse response of a venue such as the one presented in figure 2, and obtain its frequency response. This response is also referred to as the transfer function of the venue, for the particular transmission path. This response is presented in figure 3.

The frequency response plot allows for a visual identification of frequencies that are being enhanced or attenuated at a particular listening position in the venue. This plot clearly shows the need for correction. The transfer function in figure 3 indicates, for instance, that at the particular listening position, there is an attenuation of over 20dB at around 600Hz, and a gain of roughly 5dB just over 100Hz. A flat response at that position would allow for the reproduction of all frequencies with the same gain.

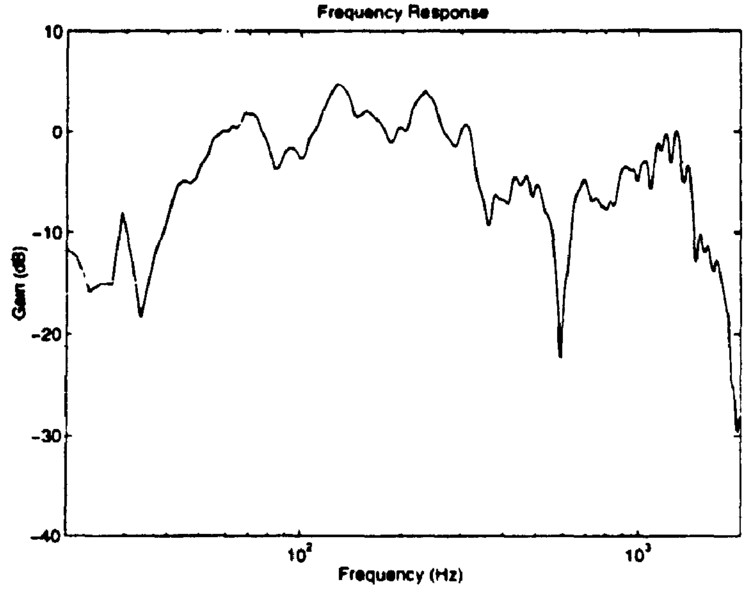


FIGURE 3. Frequency Response of a venue (Fourier Transform of the impulse response in figure 2)

The frequencies represented in figure 3 range up to 2KHz, as the impulse response was collected with a sampling frequency of 4KHz.

Spatial Variation of the Transfer Function

The number of independent transfer functions of a room measured at different listening positions is a function of the wavelength [Cook55][Schr62][Bena85]. In this section we will show how this variation of the transfer function has to be considered when choosing an upper frequency limit for the equalization. For illustration purposes, table 1 shows frequencies and wavelengths within the audible spectrum, and will be used as reference for some examples. Values were calculated with speed of sound at room temperature being 343m/s.

In order to characterize the behaviour of the sound field in a venue, one needs to measure the transfer functions of the venue from the sound source to different listening positions. If the response of the room is measured at two points exactly adjacent to one another, the same reading will be collected. For a certain frequency, the measurements can be considered statistically independent for distances greater than a fraction of the wavelength. It has been shown [Cook55][Bena85] that this fraction is at least one half of the wavelength. This means that for distances smaller than one half of the wavelength, statistical similarities between the readings can be measured, for instance, by a correlation function.

TABLE 1. Frequencies and Wavelengths

Frequency	Wavelength
20 Hz	17.15 m
50 Hz	6.86 m
100 Hz	3.43 m
300 Hz	1.143 m
500 Hz	68.6 cm
1000 Hz	34.3 cm
5000 Hz	6.86 cm
10000 Hz	3.43 cm
20000 Hz	1.71 cm

From the previous statement, one can conclude that to evaluate the sound field of a room for higher frequencies (shorter wavelengths), one needs to measure the transfer functions at points that are just centimetres apart. This would be virtually impossible to be achieved. For lower frequencies, the response measured at a smaller number of points can be considered as characterizing the sound field of the room. Finally, we can state that for low frequencies (say, below 500Hz) the transfer function measured at a particular listening position can be considered to represent the room response up to about 30cm away.

A more formal way of presenting the problem of evaluating the sound field perceived by the listener at a certain position within a venue, takes into account the volume enclosed within the boundaries of that venue, and its vibrational modes [Bera54] [B&K78]. Let us illustrate the problem with a standard example: a rectangular room. For rectangular rooms, the modes determine planes of vanishing sound pressure. For real rooms, however, these regions of vanishing sound pressure are no longer planes, but surfaces. Technically they are referred to as "nodal surfaces" [Kutruf91].

Let us consider only the simplest geometrical configuration, or a room of dimensions L_x=2.0m, L_y=1.2m, and L_z=1.5m. These are roughly the dimensions of the interior of a car, and were chosen to make the example consistent with the previous explanation. The number of "lattice points", or points of "vanishing sound pressure", due to the resonance modes in this hypothetical venue can be calculated using the following equation [Morse48]:

$$N_f = \frac{4\pi}{3} \cdot V\left(\frac{f}{c}\right)^3 + \frac{\pi}{4} \cdot S\left(\frac{f}{c}\right)^2 + \frac{L}{8} \cdot \frac{f}{c} \qquad \text{(EQ 5)}$$

Where V is the volume of the venue, S is the sum of the area of all walls and L the sum of all edge lengths. A table can then be constructed for different frequencies:

TABLE 2. Frequencies and number of lattice points

frequency	Number of points
100 Hz	2
500 Hz	74
1000 Hz	476
5000 Hz	49148

As expected, the number of lattice points increases with an increase in frequency. This naturally implies that in order to have an accurate evaluation of the sound field within the venue up to high frequencies, the responses at a great number of points should be measured.

The use of a rectangular room is a very simple approximation, and there is still a great deal of mathematical structure that could be exploited. That, however, is made impossible by the very complex and unknown geometry of real rooms. We use the rectangular room only as an illustration of the aspects involved with the spatial variation of the room transfer function. In order to visualize the problem in a better way, a plot simulating the tangential mode of a room with a standing wave is presented in figure 4.

The plot shows in the top graph the standing wave with high and low sound pressure zones. The example represents, for instance, a room of dimensions 2 m x 1.5 m, and the tangential mode for a frequency of 343 Hz ($\lambda = 1$m). Since only the tangential mode is shown, only the reflections from wall to wall (between the four walls) are taken into account. Reflections from floor and ceiling added to this mode would constitute the oblique mode, which will not be described here.

The label "H" indicates high sound pressure zones. High and low pressure zones are half a wavelength apart. This implies that the transfer function measured from a sound source to two listening positions less than half a wavelength apart have some statistical similarity, for frequencies up to the one used. The top view (plane) of the sound pressure distribution plot shows with straight lines the vanishing sound pressure planes. Finally the bottom graph shows the gradient, with arrows pointing towards these zones.

If other frequencies are added, the high and low pressure zones will be spaced differently. Since the overall pattern is a function of both position and frequency, that pattern will be very complex if a signal composed of many frequencies travels in the room.

For the equalization of the overall transfer function at multiple listening positions, one is interested in equalizing a region where the listener will be located. One needs, therefore, to choose an upper bound in frequency for the equalization. This upper bound should have a big enough wavelength for the transfer function equalized to be representative of the transfer function of a region where the listener can be "fit into", around that position

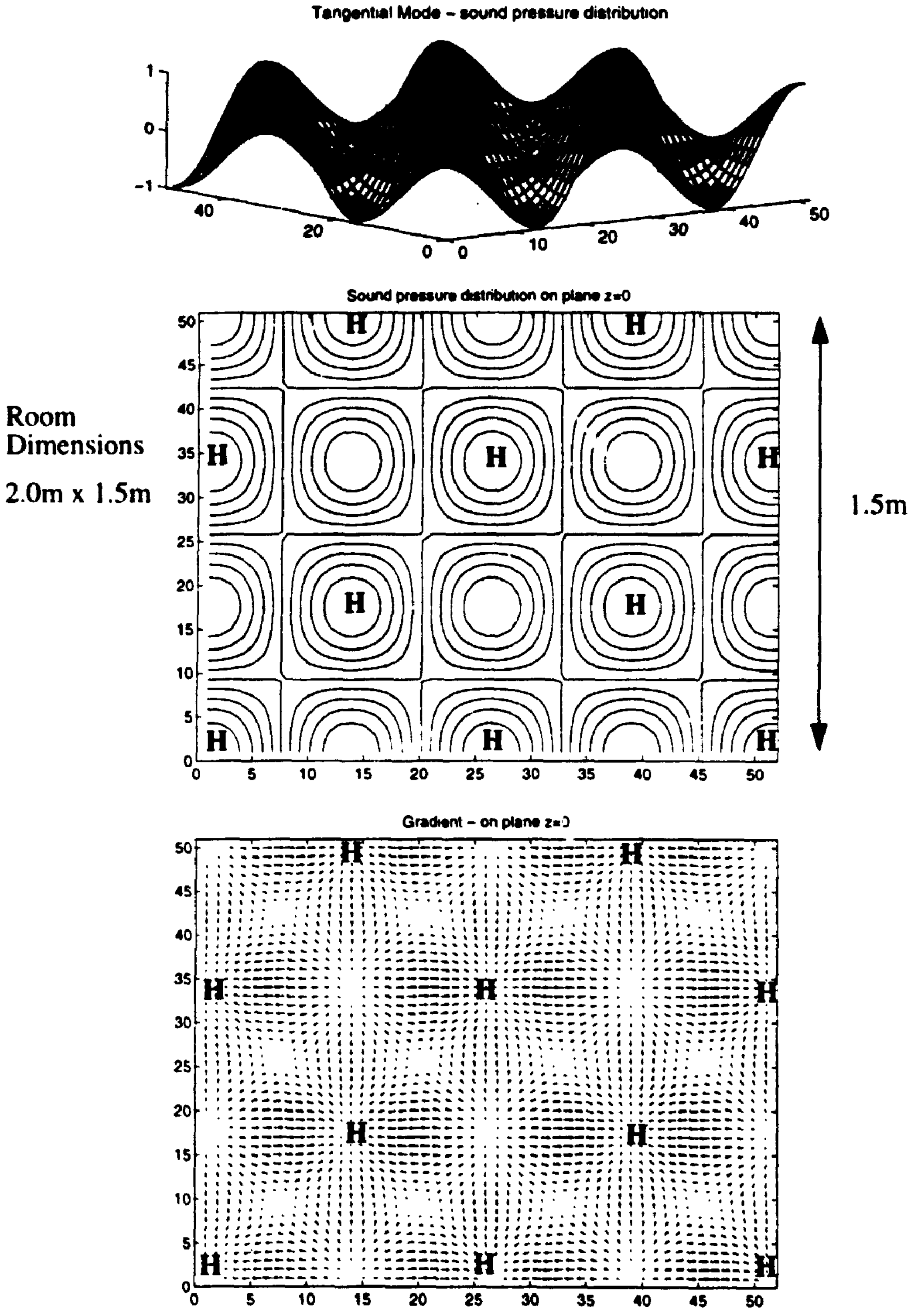


FIGURE 4. Simulation of the tangential mode in a room

where the initial measurement was taken. We have chosen 500Hz (λ=68cm) as the upper frequency limit in this work.

The main conclusion drawn from the illustration of the spatial variation of the transfer function of a room at different listening positions is that the transfer function around a certain position can roughly be considered constant up to a distance proportional to the wavelength of the highest test frequency. The distance should be less than half of the wavelength.

An upper bound for the equalization process is therefore chosen to be 500Hz, which will allow for the equalization of a region of approximately a third of a metre around the listener- approximately the size of a human head.

Room Acoustics - Psychoacoustic Aspects

The field of psychoacoustics studies the ear and its response to different frequencies, as well as the most relevant characteristics present in the acoustics of the room as they are perceived by the human ear. The most intriguing aspect of psychoacoustics is related to the "psycho" portion of the study, rather than to the "acoustics". The entire field of research is based on subjective measurements. The results presented need to be interpreted with special care, since none of them was measured by an instrument. Rather, a listener indicated a preferred situation.

In the next paragraphs, we will present a brief overview of research done on the filtering abilities of the ear, and on the audibility of features present in the acoustics of a venue.

Although the study of the influence of the surroundings on the listener have been developed since the early part of this century, the early research on psychoacoustics focused on understanding the filtering characteristics of the ear. Fletcher [Harr69] was primarily concerned with the pattern of excitation on the basilar membrane, and did not determine the characteristics of the auditory filter in detail. Zwicker [Harr69][Zwic61] researched the critical bands, or *Frequenzgruppe*, and their application to loudness.

A more thorough investigation on the audibility of frequency response irregularities had not been developed until Bücklein [Bück62] published a research paper in 1962. The conclusions drawn in his publication are the reasons why we are interested in equalizing frequency responses. We shall comment on his conclusions later in this section.

Equalization for low frequencies is of particular interest due - thus far - to the physical constraint imposed by the size of the wavelength. Research has also been done specifically in the audibility of low frequencies.

Fidell [Fide83] confirmed with measurements made up to 100Hz that "head movements through low-frequency standing waves did not affect signal levels by more than 1dB." This reinforces our previous statement about the variation of room transfer function. At 100Hz, the transfer function can be considered similar to one measured a metre away.

More recently, Fincham [Finc85] investigated the subjective importance of uniform group delay at low frequencies. He focused on the recording/reproduction chain presenting phase distortion at around 40/50Hz, and noticed that effects could be audible even in male speech, which has little energy below 100Hz. He concludes that "group delay distortion at low frequencies can reproduce subtle but clearly audible changes in sound quality". For this reason we equalize for linear phase - constant group delay.

A research paper by Lipshitz et al. [Liptz82] addresses the problem of the audibility of phase distortion in audio systems. This paper also briefly comments on phase equalization. The authors base the measurements in the range between 100Hz and 3KHz where "most researchers" have found that the ear's sensitivity to phase distortion is the greatest. The results of the listening tests confirmed a "half-wave rectifier" model of human hearing mentioned by previous researchers, reinforcing the suspicion that this behaviour of the ear would have "significant implications for the ability of the ear to detect waveform and/or phase effects at the lower frequencies." Their tests showed however, that the audibility of midrange phase distortion is much greater on headphones than on loudspeakers in a normally reverberant venue, although some of the effects were audible in both cases. A tutorial review was published by Preis [Preis82], also addressing "Phase Distortion and Phase Equalization in Audio Signal Processing". This work reinforces our interest in equalizing phase.

Toole has published many papers on subjective measurements for loudspeaker quality assessment and performance of listeners [Tool85] [Tool86a] [Tool86b], and one on general aspects of listening tests [Tool82]. These publications are extremely rich in the description of the techniques employed in the measurements, and are a great source of reference in the psychoacoustics field. From these publications, one can infer the importance of the choice of loudspeaker to be used in a system, since a bad choice could ruin the entire equalization process. He also co-authored a paper on the perception of live and reproduced sounds [Olive89]. His work confirms the importance of flat frequency response for high-quality sound reproduction, and quantifies it.

All research in psychoacoustics relies on subjective measurements. A paper by Lipshitz and Vanderkooy [Liptz81] presents a good debate on subjective evaluation. This paper together with the papers by Toole (that includes [Tool82]) will provide the reader with a very good understanding of the meaning and application of listening tests.

The psychoacoustic aspects of room acoustics will all be related to the audibility of features present in the transfer function of a venue. From the psychoacoustics point of view, there are some main reasons why one should be concerned about the acoustics (or in other words: the transfer function) of a room. These are:

1. Peaks and notches in the transfer function are audible. Peaks are far more audible and annoying than notches of the same intensity for a certain frequency [Bück62]. A notch in the transfer function at a certain location of a room could be caused, for example, by a hard reflection of the sound on a nearby surface. Figure 4 illustrates the case where a reflected portion of the signal interferes with the signal on the direct path.

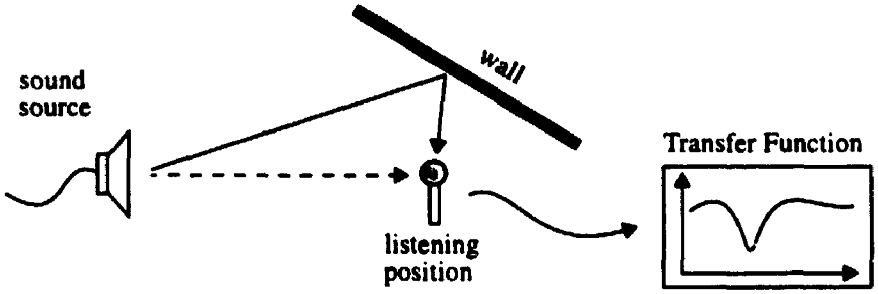


FIGURE 5. A hard reflection causing irregularities on the transfer function

Bücklein's publication showed basically that peaks in the frequency response are "far more audible" than valleys or dips. He went further to show that in transmission systems, peaks in the transfer function reduce intelligibility more than corresponding valleys. Bücklein also noted at some point of his work that for low frequencies (300Hz) all observers detected a 5dB valley in the transfer function of the system presented. He noted, however, that "valleys, even when clearly detectable, do not change the sound quality as much as equally large peaks, which can appear very unpleasant". These results were achieved using speech and music as test signals, called by him "natural sounds". Another interesting result reported was the detection of a 10dB peak at 90Hz by a larger number of listeners than a 25dB valley at the same frequency. In summary, Bücklein states that in order to obtain a reasonable (satisfactory) transmission, one can tolerate small (even deep) valleys, but must avoid narrow peaks.

2. The ear can be considered as a collection of very complex filters that vary in shape according to the stimulus applied to the ear [Patt74][Patt77]. This is a concern when low frequency signals mask high frequency ones [Eve86]. Higher frequency signals are vital for speech intelligibility [Davis89]. The transfer function of a room should keep the transmission path from source to listening position free of distortion.

3. For the average listener, it is possible to detect a difference in sound pressure level of about 1dB for any tone between 50 and 10KHz, if the level of the tone is greater than 50 dB above the hearing threshold for that tone [Bera54]. For levels below 40dB, the perceptible level differences increase to 3dB. This implies that small differences in the overall gain at particular listening positions may not be bothersome to listeners.

4. Flat delays of a few milliseconds are acceptable for recorded music, and the overall gain (or "volume") perceived by a listener can vary quite widely without affecting quality. These are the degrees of freedom we exploit in the next chapter.

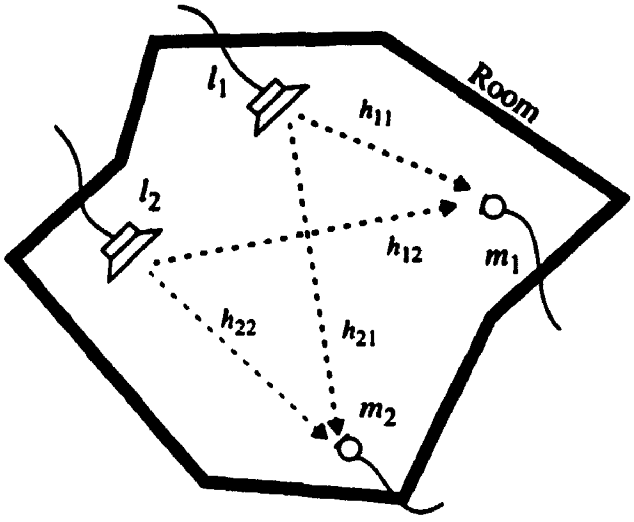
With the physical and psychoacoustic aspects presented, the meaning of equalizing the overall transfer function of a room will be presented in the next section.

Room Equalization

The problems involved in equalizing the transfer function of a room at a listening position have been presented in chapter 1. In the next sections, the equalization is presented for more than one listening position, and the problem is approached using basic concepts of control theory and linear algebra.

A System Called "Room"

Let us consider a system with two sound sources and two listening positions as presented in figure 6. This system will need four transfer functions to be characterized, from each sound source to each listening position.



l_1 and l_2 are sound sources

m_1 and m_2 are listening positions

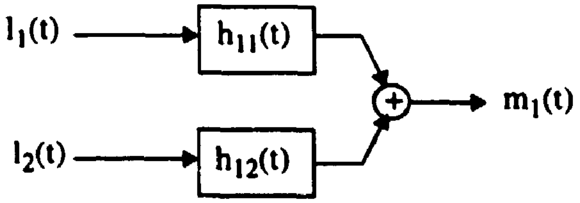
h_{11}, h_{12} are transfer functions from sources 1 and 2 to position 1

h_{21}, h_{22} are transfer functions from sources 1 and 2 to position 2

FIGURE 6. A simple 2x2 system

The above 2x2 system can be described in the following way:

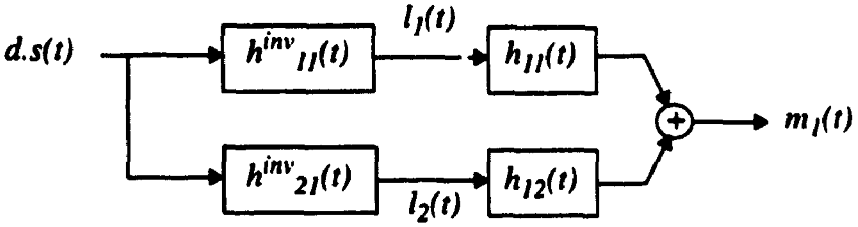
In the time domain, for each listening position, one will have a system corresponding to the block diagram (formulation corresponding to the first listening position only):



This would correspond to the equations, for each listening position:

$$m_1(t) = h_{11}(t) \otimes l_1(t) + h_{12}(t) \otimes l_2(t)$$
$$m_2(t) = h_{21}(t) \otimes l_1(t) + h_{22}(t) \otimes l_2(t)$$
(EQ 6)

In order to achieve equalization for the system described, assuming the same signal is to be sent to the loudspeakers, one could implement a system according to the following diagram (shown for position 1 only):



In this case, "*s*" represents the signal with desired gain "*d*" going to the equalizer; the "inverse filters" represent the equalizer and finally "*l*" represents the signal that is actually emitted through the loudspeakers.

This block diagram would then correspond to the equation (shown for first position only):

$$m_1(t) = h_{11}(t) \otimes h^{inv}{}_{11}(t) \otimes d \cdot s(t) + h_{12}(t) \otimes h^{inv}{}_{21} \otimes d \cdot s(t)$$
$$m_1(t) = 2d \cdot s(t)$$
(EQ 7)

Equalization - Algebraic Approach

Since the algorithm to be presented in the next chapter is heavily based on concepts found in linear algebra, this section presents a review of those concepts and links these concepts to the multipoint room equalization problem.

Following the results found in the analysis just presented, an algebraic formulation is developed using the same system presented in figure 6. With this approach, listening positions as well as sound sources will compose vectors, and the transfer functions involved will be placed in a matrix of transfer functions.

Equations 1 and 2 can be re-written using a matrix notation:

$$[m(t)] = [h(t)] \otimes [l(t)] \qquad \text{(EQ 8)}$$

From equation 3, and using the Fourier transform, one can establish the following relation:

$$\begin{bmatrix} m_1(t) \\ m_2(t) \end{bmatrix} = \begin{bmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{bmatrix} \otimes \begin{bmatrix} l_1(t) \\ l_2(t) \end{bmatrix}$$

$$\updownarrow \mathcal{F} \qquad \text{(EQ 9)}$$

$$\begin{bmatrix} M_1(\omega) \\ M_2(\omega) \end{bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix} \cdot \begin{bmatrix} L_1(\omega) \\ L_2(\omega) \end{bmatrix}$$

From this point on, we will use the frequency domain notation. In order to simplify the formulation, the vector of signals received at listening positions will be called vector M, the matrix of transfer functions will be called matrix H, and vector L will represent the signals emitted through the sound sources.

To eliminate the influence of the room on the sound from sources l_1 and l_2 arriving at the listening positions, it suffices to find an inverse of matrix H and apply it to the signal prior to sending it to the speakers. Vector L could then be substituted by a vector of gains that are to be applied - if necessary - to the signal, represented by a scalar S. Reformulating the equations in the frequency domain and adding the equalizer to the system, one will have:

$$[M(\omega)] = [H(\omega)] \cdot [H(\omega)]^{-1} \cdot [d] \cdot S(\omega) \qquad \text{(EQ 10)}$$

And therefore:

$$[M(\omega)] = [d] \cdot S(\omega) \qquad \text{(EQ 11)}$$

This is the basis of the approach for the acoustical equalization of a venue at multiple positions. The inverse to be found is not anymore the inverse of the each independent transfer function, but rather the inverse of the matrix of transfer functions.

Review of Matrix Algebra

For the multipoint equalization approach to be presented, it is very important to review some concepts of matrix algebra. These concepts will further be used in the analysis of data collected in multiple positions within a room. This section reviews only two basic topics: inversion and eigendecomposition of a matrix. The following review is based on [Noble69][Boldr80][ONeil83].

- Matrix Inversion

From Linear Algebra, the definition of the inverse of a matrix is that given a square matrix A of order n, a matrix B can be called the inverse of A if

$$B \cdot A = A \cdot B = I_n \qquad \text{(EQ 12)}$$

where I_n is the identity matrix of order n. If A admits an inverse, it is called nonsingular. Matrix B can also be represented by A^{-1}. Some features present in matrix A to be inverted can indicate if it is in fact invertible. For instance, if all rows and columns of matrix A are linearly independent, the matrix admits inversion. In the next section, the inversion will be analyzed with eigenvalues and eigenvectors.

If another nonsingular matrix C is given, then the following relations are valid:

$$(AC)^{-1} = C^{-1}A^{-1} \tag{EQ 13}$$

- Eigenvectors and Eigenvalues

Given a nonsingular square matrix A, of order n, an eigenvalue λ and an eigenvector v are solutions for the equation:

$$A \cdot v = \lambda v \qquad \text{or} \qquad (A - \lambda I)\, v = 0 \tag{EQ 14}$$

where I is the identity matrix of same order n.

If we now use a matrix of transfer functions H such as the one presented in equation 10, we see that the identity matrix resulting from the equalization process is a diagonal matrix with the elements of the main diagonal - scalar gains - equal to "one" at all frequencies, as it was expected. It is also possible to define H in terms of its eigenvalues and eigenvectors. Let matrix V be a matrix with the eigenvectors of H, and matrix D a diagonal matrix with the eigenvalues of H corresponding to the eigenvectors in V, such that the i^{th} column (eigenvector) in V corresponds to eigenvalue i in D. V is always full-rank, while D and H may be singular. We can decompose H as:

$$H(\omega) = V(\omega) \cdot D(\omega) \cdot V^{-1}(\omega) \tag{EQ 15}$$

Consider, for illustration, that for a particular frequency there is a scalar "gain" (eigenvalue of H) equal to zero. Matrix H will not be invertible (i.e. it is singular), since there is an input (the eigenvector corresponding to that eigenvalue) that will produce zero output for that frequency. Therefore, if an eigenvalue is zero for a matrix, that matrix will be singular.

Similarly to equation 15, the inverse of a matrix H can be written in terms of the eigenvalue and eigenvector matrices of H:

$$H^{-1}(\omega) = V(\omega) \cdot D^{-1}(\omega) \cdot V^{-1}(\omega) \qquad \text{(EQ 16)}$$

From this equation one can confirm that if an eigenvalue of H is zero, H is singular. The eigenvalue matrix D is a diagonal matrix, and the inverse of a diagonal matrix is another diagonal matrix with the reciprocal of each element of D as its non zero elements. Therefore, if one of the elements of D is zero, the inverse of D cannot be calculated. As a consequence, the inverse of H also cannot be calculated.

In the next chapter, the formulation of the acoustical room equalization problem will be extended using the concepts just reviewed. At that point, the physical meanings of the transfer function matrix and its eigendecomposition will be derived in further detail, as well as the role played by the drivers (sound sources) and the equalizer.

Background work

As has been shown earlier in the chapter, the whole idea behind room acoustical equalization is to perform inverse filtering in a room to achieve a flat frequency response of the room over the audible frequency band. In this section we review the previous research done in room acoustics, invertibility of room transfer functions and room equalization.

Roughly speaking, after investigating the development of research on room acoustics, we notice that studies on the room transfer function [Schr54] [Lyon69][Tohy88][Mourj91] lead to studies on dereverberation of speech and musical signals [Allen77] [Nomu91] [Nomu93]. On the other hand, these lead to further research on the invertibility of room transfer functions which finally lead to single and multiple point room acoustical equalization.

The pioneers on the investigation of the invertibility of room transfer function were Neely and Allen [Neely79], who used the Nyquist stability criterion to analyze room transfer functions. Impulse responses were synthesized as being pure delay plus a minimum phase component or delay plus a non-minimum phase component. They attempted to equalize both types with the inverse of the minimum phase component only (the inverse of a non-minimum phase system is unstable). The system with delay plus minimum phase component was equalized without problems, but the attempt to equalize the transfer function that had a non-minimum phase component with the same filter resulted in a constant audible "tone", that was proven to be the all-pass component of the room that could not be removed by inverse filtering. They conclude from simulation that a typical room "would have a non-minimum phase effect on a speech signal when the receiver is more than 8 inches from the source". Although this seems to be discouraging, Genereux [Genrx93] implemented a system dealing with the non-minimum component of the transfer function of a real room.

Mourjopoulos [Mourj82][Mourj85] first compared homomorphic methods (breaking the non-minimum phase sequence - or response - into minimum and maximum phase components prior to inversion) and least squares (linear predictive) methods in order to achieve the inversion of mixed-phase sequences, such as an impulse response of a room. The performance of the least squares methods were usually observed to be better. Mourjopoulos was the first to report the degradation of the transfer function at one listening position due to an attempt to equalize the transfer function at another listening position in a venue [Mourj85].

As it has already been mentioned, in 1982 Lipshitz et al [Liptz82] published a paper dealing with some aspects of the equalization of non-minimum phase systems. However, that publication deals strictly with the audibility of phase distortion. Later in the same year, Preis [Preis82] published a tutorial review dealing with phase distortion and equalization for audio signal processing.

The majority of the papers approaching the equalization problem, employ adaptive time domain systems. Such an approach will involve dealing with problems such as causality, long filters and variability of room transfer function to be inverted. Two of those, however, [Mourj94][Munshi90] make use of different techniques to achieve equalization.

Mourjopoulos [Mourj94] made use of vector quantization techniques associated with an all-pole model of the room response to achieve room transfer function equalization. The all-pole model [Mourj91] of the transfer function can achieve a reduction in the filter length requirement. The vector quantization groups the transfer functions in order for those to be equalized and achieve spectral flattening over a region within the venue. He pointed out that if a listening position can be specified, the complexity of this approach can be greatly reduced in a practical implementation.

Munshi [Munshi90] makes use of multiple loudspeakers to equalize multiple listening positions. He presented a frequency domain adaptive algorithm to correct the frequency response of a room at several listening positions. Munshi looks for an inverse filter for each sound source used, adaptively, using the frequency domain information collected at each listening position (which contain information on the overall transfer function at that position). The total frequency domain error was used to update the adaptive filters. He also presents an algebraic interpretation of the room equalization problem, by analyzing the rank of a matrix of transfer functions.

The main differences between Munshi's approach and the one presented in this thesis are a) the adaptive algorithm he used, to find approximate inverses for the overall transfer functions at each location, and b) he does not identify *a priori* where in frequency a problematic inversion will occur. In this thesis we identify the problematic frequencies first and find an approximate solution for the inverse filters only for those frequencies. For all other frequencies within the band of interest, the inverse filters are guaranteed to be exact inverses of the overall transfer function at each position.

Since his approach makes use of an FFT-based algorithm, blocks of data have to be collected to perform the adaptation. That in a real-time implementation could cause a noticeable latency problem. From his results, one can see that he handles singularities of the transfer function matrix to a certain point, by finding approximate solutions for the inverse filters at all frequencies.

Different solutions are described by [Neely79][Mourj85] for the inversion of non-minimum phase systems, while [Genrx93] describes a generalization of the previous ones. Genereux proposed an adaptive system making use of a linear predictor with a "segmented" type of FIR filter. The filter was called "segmented" due to the segmentation of its impulse response into bandlimited time intervals. The interesting feature pointed out by Genereux is the reduced frequency resolution for the mid and the high frequency portions of the audible spectrum, in order to "minimize location specificity".

Miyoshi and Kaneda [Miyo86] set extra "signal-transmission channels" to achieve equalization of one position in a venue. In other words, they added loudspeakers and microphones providing more transfer functions for the equalizer to work with. The algorithm achieves adaptively the "exact" inverse of a non-minimum phase system. Compared to models based on least squares error (LSE), the performance of the new approach was found to be better. They also repeated the results in [Miyo88] and extended the principle to multiple points, without presenting results.

Elliott et al. [Elliott87][Elliott89][Elliott94] first introduced a multiple error LMS algorithm and tested for the inverse filtering of a room. Later in 1989, they presented results using a similar system, pointing out that there is improvement in the transfer function of one position, but - as expected - degradation elsewhere. The same configuration was tested for multiple positions without relevant improvement. In 1994, they pointed out that the best filtering strategy for a single point equalization appeared to be a weighted multiple point equalization, in which the error at a certain position was more heavily weighted in the adaptation algorithm, and that would avoid the degradation in other positions. The equalization was performed below 400Hz.

Elliott was also the first to publish research on the acoustical equalization of a room at multiple points[Elliott89] [Elliott94], making use of an adaptive single-channel multiple-error LMS formulation [Elliott87]. He first implemented the equalizer by adjusting the filter coefficients in order to minimize the sum of the squares of the errors between the equalized responses at the listening locations and delayed versions of the original signal. Then, in [Elliott94] he tried the same technique with a slightly different system, without achieving much improvement. The technique was tried with success however for single point equalization. Elliott noted that "the only way in which more significant improvement can be obtained in the equalized response at all four microphones is to use multiple loudspeakers and multiple equalization filters." Studies on adaptive systems using multiple loudspeakers were done in [Elliott94b], but focused on active control, rather than room equalization in particular.

At the time this thesis was being prepared, a paper by Wang [Wang95] was presented, making use of Padé approximation for multi-channel deconvolution. The ratio of two transfer functions is approximated by a polynomial, by estimating one reverberant signal from another using least mean squares. The transfer functions are then estimated using Padé approximation. No comparison was presented with previous research, and results from simulation were commented without much detail.

The main streams on room acoustical equalization can be divided in two, namely: for a single point in a venue with one or multiple drivers; and for multiple points in a venue, with one or multiple drivers. This thesis belongs in the group that uses multiple drivers to achieve overall acoustical equalization at multiple locations in a room.

Conclusion

A flat overall amplitude response is the first concern in equalizing the transfer function of a room at one or several listening positions. A flat group delay is desirable, if possible. The literature review reinforces these two assumptions.

It is a difficult task, however, to equalize the room response for higher frequencies due to factors such as the variability of the transfer function. Once again, previous research has shown that acoustic equalization is effectively performed only at low frequencies.

For that reason, we have limited the algorithm to focus on the region below 500Hz. Above this region, other criteria are applied, since the direction of arrival and the influence of early reflections for instance have to be taken into account.

CHAPTER 3

A Novel Multipoint Equalization Algorithm

Introduction

This chapter presents a novel algorithm for the equalization of the overall transfer functions of a room, at multiple listening positions. This algorithm allows for a shift in the solution of the inverse filtering problem, from inverting each independent transfer function to finding a suitable inverse for a matrix of transfer functions. The transfer functions represent the multiple paths from sound sources to listening positions, and the resulting matrix is called the "transfer function matrix H".

The algorithm to be presented systematically identifies the frequencies where problems are detected for the inverse filtering process. At these frequencies the transfer function matrix H is difficult - if not impossible - to invert.

In the new equalization algorithm, inverse filtering is achieved by allowing adjustments of signal gains in conjunction with a new pseudo-inverse of the transfer function matrix, in order to emulate the exact inverse of the matrix for the particular gains chosen. The desired set of gains will be calculated first for problematic frequencies and then applied to the other frequencies of the band of interest. A new inverse of matrix H is calculated for each case, at each frequency. Both the desired set of gains and the new inverse of H are calculated based on the eigendecomposition of H for problematic frequencies found within the band of interest.

Numerical explanations are given with simple examples, where they are considered necessary. Physical meanings are also given for each step of the process, in order to create a parallel view of the otherwise numerical-only approach.

Throughout this chapter, at each frequency, matrix V denotes the matrix of eigenvectors of H, and matrix D the diagonal matrix of eigenvalues of H.

What Exactly One is Searching For

It has been mentioned that the focus of solving the equalization problem has been shifted from the search for an inverse for each of the independent transfer functions to the search of an inverse for a matrix of transfer functions.

At some frequencies, the transfer function matrix will present nearly singular behaviour, indicating a difficult inverse tor that matrix. Identifying the frequencies where matrix H presents a difficult inverse is the first step of the equalization process.

What is a difficult inverse? Let us consider two small 2x2 matrices, one (example A) with the two rows being "almost" linearly dependent and the other (B) with the rows being linearly independent. Both represent matrix H for hypothetical frequencies. Using Matlab [Matlab] code, one can verify that the inverse in example A contains much larger values

Example A)

```
>> H=[3 5; 6 9.99]

H =  3.0000 5.0000
     6.0000 9.9900

>> I=inv(H)

I =  -333.0000 166.6667
     200.0000 -100.0000
```

Example B)

```
>> H2=[3 5; 6 15]

H2 =  3.0000 5.0000
      6.0000 15.0000

>> I2=inv(H2)

I2 =  1.0000 -0.3333
      -0.4000 0.2000
```

If one is interested in performing inverse filtering using the elements of these inverses of H at each frequency, example A would represent a high gain peak at the frequency where H is nearly singular. Such a characteristic is undesirable. Though the correction in example A would be achieved at the listening positions involved, very high peaks would occur somewhere else in the venue. We are searching for a way of achieving the correction while avoiding high gain peaks at other positions in the venue.

At this point, it is known how to construct matrix H for every frequency with measured transfer functions. It is also known that this matrix can be decomposed into matrices V (of eigenvectors) and D (of eigenvalues). If one now breaks the examples A and B into eigenvalues and eigenvectors of H and H2, it is clear how to identify the frequencies with difficult inverse of the transfer function matrix through eigendecomposition:

Example C)

```
>> H= [3 5 ; 6 9.99]

H =  3.0000 5.0000
     6.0000 9.9900

>> [V,D]= eig (H)

V =  -0.8573 -0.4475
      0.5148 -0.8943

D =  -0.0023 0.0000
      0.0000 12.9923
```

Example D)

```
>> H2= [3 5 ; 6 15]

H2 = 3.0000 5.0000
     6.0000 15.0000

>> [V2,D2]= eig (H2)

V2 = -0.9204 -0.3337
      0.3910 -0.9427

D2 =  0.8760 0.0000
      0.0000 17.1240
```

A matrix with a zero value as one of the eigenvalues does not admit an inverse. Similarly, a matrix with very small (close to zero) eigenvalues will present a problematic inverse.

Physically speaking, if a vector having the same entries as the eigenvector associated with the biggest eigenvalue is "input" to matrix H, the "output" will be that vector scaled by the biggest eigenvalue. Generalizing for any non-singular matrix, an "input" to the matrix equal to one of its eigenvectors will produce an "output" equal to that vector scaled by the corresponding eigenvalue.

For the very small eigenvalue, there is an input that will produce a nearly-zero output, and therefore one will clearly have a problem at that frequency regarding what can or cannot be done in terms of inputs and outputs. The eigenvector corresponding to that very small eigenvalue corresponds to something that cannot easily be done by the system at the frequency presenting an ill-conditioned matrix H.

One needs to find a way of overcoming such a problem, since it could occur when a very small eigenvalue is detected for H at one or more frequencies within the band of interest. A feasible and efficient method of identifying and fixing the inversion of matrix H at problematic frequencies is to find a suitable substitution for the inverse of H at the frequencies where a difficult inverse is detected.

A Novel Multipoint Equalization Algorithm

We defined H as a rectangular matrix with rows indicating the listening (microphone) positions and the columns indicating the source (loudspeaker) positions. For each discrete frequency, one transfer function matrix is obtained. From this point on, matrix H is always taken to be a square matrix - for simplicity - so the number of listening positions equals the number of sound sources. The general case is left for future work.

Frequencies presenting a very small eigenvalue in the transfer function matrix will have a problematic inverse. The inversion of matrix H needs to be somehow performed for these frequencies, in order to achieve equalization at multiple positions.

We shall now present a formulation for the algorithm as it was introduced, making use of a new inverse of H and a set of desired gains. This is done in equation 17.

$$\underbrace{[M(\omega)]}_{\substack{\text{Signal}\\ \text{Received}\\ \text{at listening}\\ \text{positions}\\ \text{(nx1)}}} = \underbrace{[H(\omega)]}_{\substack{\text{Transfer}\\ \text{Function}\\ \text{Matrix}\\ \text{(nxn)}}} \cdot \underbrace{\left[H_{NIH}^{-1}(\omega)\right] \cdot [d]}_{\substack{\text{Equalizer:}\\ \text{New Inverse of H}\\ \text{times a set of}\\ \text{desired gains}\\ \text{(nxn x nx1)}}} \cdot \underbrace{s(\omega)}_{\substack{\text{Signal to be}\\ \text{Equalized}}} \qquad \text{(EQ 17)}$$

The column-matrix M represents the signals received at the listening positions. Matrix H is the transfer function matrix with the values of each transmission path from sound source to listening position for each frequency. The inverse matrix H^{-1}_{NIH} and vector d represent the equalizer. H^{-1}_{NIH}, which we develop below, is an inverse of H only for a special class of gains d. For all cases presented in this thesis, only one signal is used. In equation 17 it is represented by the scalar $s(\omega)$. The general case is again left for future work.

Since there is a vector (column-matrix) of desired gains in the equalizer, a difference in the overall gain can be allowed at each listening position. From the previous chapter one knows that the ear is sensitive to peaks and notches in the transfer function, but flat transfer functions with differences in the overall gain at each location will not necessarily be audible (and bothersome) as much as a peak or a notch. In terms of equation 17, we are free to modify d.

As was mentioned in the introductory section, the first step of the equalization algorithm is to identify frequencies where the transfer function matrix cannot be inverted or will have a difficult inversion. This is done using a mathematical tool called the condition number. This number is analyzed over the band of interest in order to systematically identify and fix the problematic frequencies.

The second and third steps are respectively to find a set of gains and a suitable new inverse of matrix H to emulate the direct inverse of H. As will be seen, a set of gains d is calculated at the problematic frequencies, since these impose constraints to the system.

That set will then be applied to all other frequencies, after the new inverse is calculated for each of these frequencies.

Analyzing the Condition Number

The condition number is the ratio between the biggest and the smallest eigenvalues of a matrix. This tool allows for the identification of a difficult matrix inversion, which happens for large valued condition numbers. In other words, a difficult inversion happens with the existence of a very small eigenvalue (close to zero) for matrix H at a particular frequency or set of frequencies.

Matrices presenting a large valued condition number are known as "ill-conditioned" matrices. For each discrete frequency, the condition number of matrix H needs to be analyzed in order for one to know at which problematic frequencies the inverse of H will need to be fixed.

This analysis is shown in figure 7 as a plot of the condition number versus frequency. For every frequency over the band of interest matrix H was put together and the condition number for these matrices calculated and plotted. Figure 7a shows data up to 2KHz, and figure 7b is an expanded view of the lower frequency range we have chosen to focus on in this thesis. These results were obtained for a system with 3 sound sources equalizing 3 listening positions. The venue is the interior of a car.

One can see from the graph that around 50Hz the transfer function matrix for this system configuration is ill-conditioned. This frequency will impose constraints on equalization. Peaks spread over frequencies that are integer multiples of each other or peaks very close to each other (part of a "cluster") could indicate that they are caused by the same physical feature in the room. This observation comes in part from the condition number plots presented in chapter 4 and appendix A.

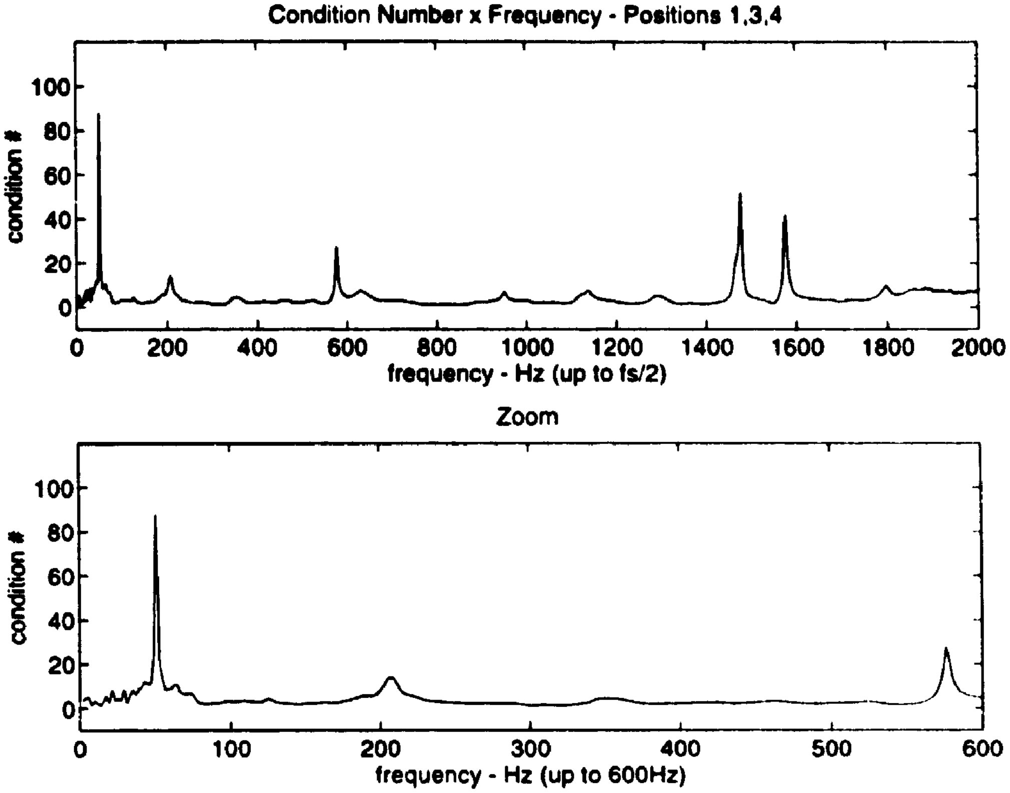


FIGURE 7. Condition Number versus Frequency

By introducing more sound sources, one also introduces new transfer functions to be used in the correction of the overall transfer function at a certain listening position. Sound sources can be seen therefore as "degrees of freedom" that could be added to the system. It is intuitively reasonable to state that the more sound sources are added to the system configuration, the more control one has over the equalization process.

There are, however, two important aspects to consider. By simply adding sound sources to the configuration, one would create a non-square transfer function matrix H, and would need to use other methods to find its inverse (or "pseudo-inverse" in that case). One of these methods is Singular Value Decomposition (SVD).

Another option is the addition of rows and columns maintaining H square. This is done by adding not only more transfer functions from existing positions to new sound sources, but also from existing sound sources to new positions and from new sound sources to new positions. In this case one would have more freedom to correct - if necessary - a bigger number of ill-conditioned matrices H, but would face the problem of satisfying all the constraints imposed by these matrices. The ill-conditioned matrices H at problematic frequencies will indicate what cannot be done with the system and will impose constraints on what can be done.

The analysis of the eigenvalue and eigenvector matrices correspondent to the ill-conditioned matrix H at problematic frequencies will determine how to find and satisfy the constraints.

What Can and Cannot Be Done With the System

Following the identification of the frequencies with ill-conditioned matrices H, one is now searching for what is possible to do with the system. In order to find a set of gains that will be applied over the entire band of interest, one needs to find gains that are possible to be imposed to the system at all frequencies.

Consider the ill-conditioned transfer function matrix H at a certain frequency shown in figure 3. The third row of H was made the sum of the two first rows.

The first desired vector, d1, indicates that one desires to have the same overall gain at each of the listening positions for every frequency. This is clearly not possible, since the transfer function matrix for that particular frequency (the problematic one) dictates that the overall gain at the third listening position must be the sum of those at the first and second positions. Adjusting the desired gains to what is possible, we obtain d2, which satisfies the constraint imposed by the problematic frequency.

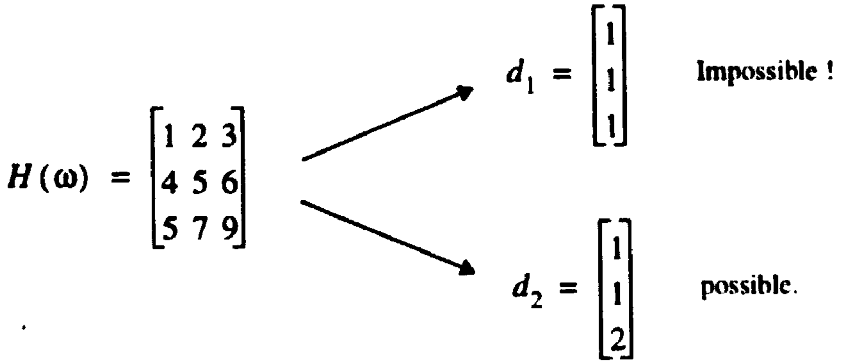


FIGURE 8. Output possible only if row3 = row2 + row1

Another problem may be the presence of two bad frequencies in the band of interest. In that case, one should look for a "good enough" desired vector that satisfies the constraints imposed by both frequencies, provided there are enough degrees of freedom. This particular detail will be clearer when a "spatial" interpretation of the problem is presented.

A Set of Gains Possible for All Frequencies

An ill-conditioned matrix H will have a very small eigenvalue (near-zero) in the *i*th-column of the eigenvalue matrix D. We will refer to it as the "bad" eigenvalue, and to the corresponding column of the eigenvector matrix V as the "bad" eigenvector.

Rearranging the eigenvector matrix V for the problematic frequency, one can place the worst eigenvector in the last column of a corrected matrix V_{corr}, and apply the Gram-Schmidt orthogonalization [Boldr80][Noble69] to obtain a new matrix V_{ort} with the orthogonalized vectors.

Physically speaking, the correction and orthogonalization are done in order for the eigenvector matrix to provide a transformation according to what can be done with the system, rather than to what cannot be done. The desired vector "*d*" should lie in the subspace spanned by the "good" vectors, and the last column of V_{ort} represents the component of an arbitrary d that must be removed.

The orthogonalization procedure can only be applied to vectors that are linearly independent. It is valid, therefore, to apply it to the eigenvectors of matrix H, since they are linearly independent. One can visualize three linearly independent vectors as vectors that are not coplanar. The visualization of such case will be clearer in a future section.

Attention must be paid, however, in application to "real world" data, since complex numbers will be involved. Due to the fact that the Gram-Schmidt orthogonalization procedure makes use of inner products of vectors in the complex space, the code (matlab or C) implementing the routines should handle that detail. The "plain" orthogonalization procedure will not work for vectors in the complex space.

The previous example introduced the restriction on what the system allows one to do (in terms of gains). That will reflect on what one desires to do. The good news, however, is that if a set of gains can be applied at the problematic frequency, at frequencies where H is well-conditioned the same set of gains can be applied. That will keep the frequency spectrum flat, for each position, over the band of interest.

A new example shall be given next. Consider the 3x3 system described by the matrix H, ill-conditioned at a certain frequency. Notice that this example resembles the 2x2 example with linearly dependent rows. Here the values on the third row are almost twice the values on the first row. The values related to the "good" eigenvalue are presented in bold face, and the ones related to the "bad" eigenvalue underlined. The eigendecomposition of matrix H and its exact inverse, the calculation of the condition number and the correction of matrix V are presented. The calculation of the desired set of gains follows the results presented next.

```
>> H=[1 2 3; 4 5 6; 2 4 5.99]

H =   1.0000 2.0000 3.0000
      4.0000 5.0000 6.0000
      2.0000 4.0000 5.9900

>> [V,D]=eig(H)

V =  0.4003 -0.3068 0.2555
     -0.8179 -0.7279 -0.8331
     0.4133 -0.6131 0.4906

D =  0.0107 0.00000 0.0000
     0.0000 11.7398 0.0000
     0.0000 0.00000 0.2395

>> Hinv=inv(H)

Hinv =  198.3333 0.6667 -100.0000
       -398.6667 -0.3333 200.0000
        200.0000 0.0000 -100.0000
```

```
>> D=eig(H)

D =  0.0107
     11.7398
     0.2395

>> CondNum=max(D)/min(D)

CondNum =   1.1005e+03

>> [Vcorr]=corrv(D,V)

Vcorr =  -0.3068 0.2555 0.4003
         -0.7279 -0.8331 -0.8179
         -0.6131 0.4906 0.4133

>> [Vort]=grashmi(Vcorr)

Vort =  -0.3068 0.3340 0.8912
        -0.7279 -0.6856 0.0063
        -0.6131 0.6468 -0.4535
```

In this example, the routines "corrv" (to rearrange matrix V) and "grashmi" (to perform the orthogonalization of the V_{corr} matrix using the Gram-Schmidt method) were written specifically to work with the multipoint equalization algorithm. These two routines are presented for the 2x2 case in Appendix C. Notice that after the correction of V, the eigenvector associated with the biggest eigenvalue (11.7398) is placed in the first column of the corrected matrix and the one associated with the smallest eigenvalue (0.0107) placed in the last column. The rearranged and orthogonalized matrix is called V_{ort}.

Some observations can be made about the example:

a) Taking a look at the two biggest eigenvalues in D, one notices that for the corresponding eigenvectors, the system responds as it should: the value at the last row of V is "almost" twice the value of the first row for the two eigenvectors associated with the biggest eigenvalues in D. In fact that pattern is observed in matrix H, and that is what makes it non invertible (rows linearly dependent). It is interesting, however, that these two eigenvectors represent what can readily be done with the system; even for that frequency, one could get twice as much overall gain at the third position than at the first.

b) The first eigenvalue in D is the problematic eigenvalue. The corresponding column in V shows that for that entry the system will not behave at all. The first column of V in this case, represents what cannot be done with the system. This entry to the system will produce an undesirable result: one will *not* be able to get with that input at that frequency twice as much overall gain at the third position than at the first, because matrix H imposes otherwise.

c) Before being rearranged and orthogonalized, the eigenvector matrix V offers in conjunction with D yet another physical interpretation of the system at that frequency. The second eigenvalue in D is much bigger than the third. The eigenvector associated with it shows that all the values are "in phase" at positions (rows) 1 and 2, rather than 180 degrees apart as they are in the third eigenvector. A bigger eigenvalue is better, and it shows it is slightly easier for the system to produce a signal in phase at all three listening positions than it is to produce them 180 degrees apart at some of the positions.

d) Considering columns of matrix V as vectors in a three dimensional space, one can say that the purpose of correcting and orthogonalizing matrix V, is to obtain a better representation in space of the eigenvectors related to the frequencies with a peak in the condition number. From this representation it will be possible to identify a region in space delimited by the good eigenvectors, isolating the bad eigenvector. This will be shown in more detail when a geometric approach is described in a future section.

It has been shown that not every vector of desired gains can be obtained. Therefore, it is necessary to transform the vector chosen to a vector that is practical. This is done using the rearranged and orthogonalized matrix V_{ort}, its inverse and a transformation matrix, as in equation 18. The new vector obtained is the closest solution one can get to the set of gains originally desired.

$$d_p = \left(V_{ort} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot V_{ort}^{-1} \right) \cdot d \qquad \text{(EQ 18)}$$

The rearranged and orthogonalized V actually converts an arbitrary *d* from what is desired to what can be done, or what will be received in terms of overall gains at the listening positions, by projecting it onto the subspace spanned by the eigenvectors corresponding to the larger eigenvalues.

Consider, therefore, matrix V_{ort} (from the previous example) and a desired vector *d*=(1,1,1):

```
>> [Vort]=grashmi(Vcorr)

Vort = -0.3068 0.3340 0.8912
       -0.7279 -0.6856 0.0063
       -0.6131 0.6468 -0.4535

>> d=[1 ; 1 ; 1]

d =  1
     1
     1
```

```
>> transM = [1 0 0 ; 0 1 0 ; 0 0 0]

transM =  1 0 0
          0 1 0
          0 0 0

>> dp = (Vort * transM * inv(Vort)) * d

dp =  0.6042
      0.9972
      1.2014
```

This new set of gains can be applied to the system at all frequencies, since what was impossible to get has been transformed into what can be provided by the system. H is well-conditioned at other frequencies and *d* is in the "good" subspace.

If a different desired vector (d2) is used, equal to the first column of Vort, no correction is made by the calculation.

```
>> d2=[ -0.3068 ; -0.7279 ; -0.6131]

d2 =  -0.3068
      -0.7279
      -0.6131

>> dpossible2 = (Vort * transM * inv(Vort)) * d2

dp2 =  -0.3068
       -0.7279
       -0.6131
```

The New Inverse of H

After finding a desired vector that can be applied for all frequencies of the band of interest, a new inverse of matrix H needs to be calculated for both "bad" and "good" frequencies.

For the problematic frequencies, this is done using the eigenvector matrix V and its inverse, and an altered version of the inverse of the eigenvalue matrix D. The altered inverse of D will have the component relative to the smallest eigenvalue forced to zero. This is shown in equation 19 where, for instance, the bad eigenvalue is located in the third column of D.

$$H^{-1}{}_{NIH} = V \cdot \begin{bmatrix} \frac{1}{D_{11}} & 0 & 0 \\ 0 & \frac{1}{D_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot V^{-1} \qquad \text{(EQ 19)}$$

For the frequencies where matrix H is well-conditioned, the new inverse is computed using both the direct inverse of H and the set of desired gains d_p , calculated with the constraints imposed by the problematic frequency. This is shown in equation 20.

so that:

$$\left[H_{NIH}{}^{-1}(\omega)\right] = \frac{\left[H^{-1}(\omega)\right][d_p][d_p]^T}{\|d_p\|^2} \qquad \textbf{(EQ 20)}$$

One has now the set of desired gains to be applied at all frequencies, the new inverses of matrix H for the problematic frequencies, and the new inversion of H for the good frequencies, making use of the direct inverse of matrix H and the desired set of gains calculated.

The Solution Seen in Space

For each of the frequencies with a peak in the condition number, the eigenvalues and eigenvectors of matrix H are calculated and used to find a desired set of gains and a new inverse for the transfer function matrix. In this section, the geometrical interpretation of the solution is presented. Since visualization is clearer in three dimensions, this description will concentrate on a system with three sound sources equalizing three listening positions, or a 3x3 system. Some details will be "reduced" to the 2x2 case.

The main objective of the equalization algorithm is to deal with the eigenvectors of H for frequencies where H has high condition number, since they will represent constraints on what can be done with the system. The solution found is then applied at all frequencies.

This can be visualized as finding a common place in the space (of the same dimension of matrix H), in which a vector will represent what can be done with the system - in terms of overall gains - at all frequencies within the band of interest. One is therefore finding where in space the desired set of gains is to be. The process of choosing this desired set of gains makes use of what is possible at the problematic frequencies and discards what is impossible.

For each frequency, matrix V_{ort} (V corrected and orthogonalized) can be seen - for a 3x3 system - as three orthogonal vectors defining a three dimensional space. Two of these vectors will define a plane and the third vector will then be orthogonal to this plane. If we consider that for each problematic frequency, the two best eigenvectors define a plane with the worst eigenvector orthogonal to that plane, for two problematic frequencies one would have two of those planes intersecting defining a vector. For this configuration, the vector of desired gains simultaneously possible at two problematic frequencies has just been determined.

A common good region in a certain n-dimensional space is obtained at the intersection of all spaces defined by the good eigenvectors of each problematic frequency. For instance, in a 2 dimensional system (two listening positions and two sound sources), for each problematic frequency there is a good eigenvector and a bad eigenvector. One should find a common region where the desired vector is to be placed, and that is at the intersection of the good eigenvectors. In this case, this region is a point: the origin.

This is an interesting result, practically speaking. The origin simply means that the desired overall gain is zero at the listening positions. This is always possible (for any system configuration), since the intersection of good regions will always encompass the origin. If one desires an overall gain of zero at the listening positions, this satisfies the constraints imposed by the problematic frequencies and is possible to be done for all other frequencies. One should just shut the system off - the "trivial solution".

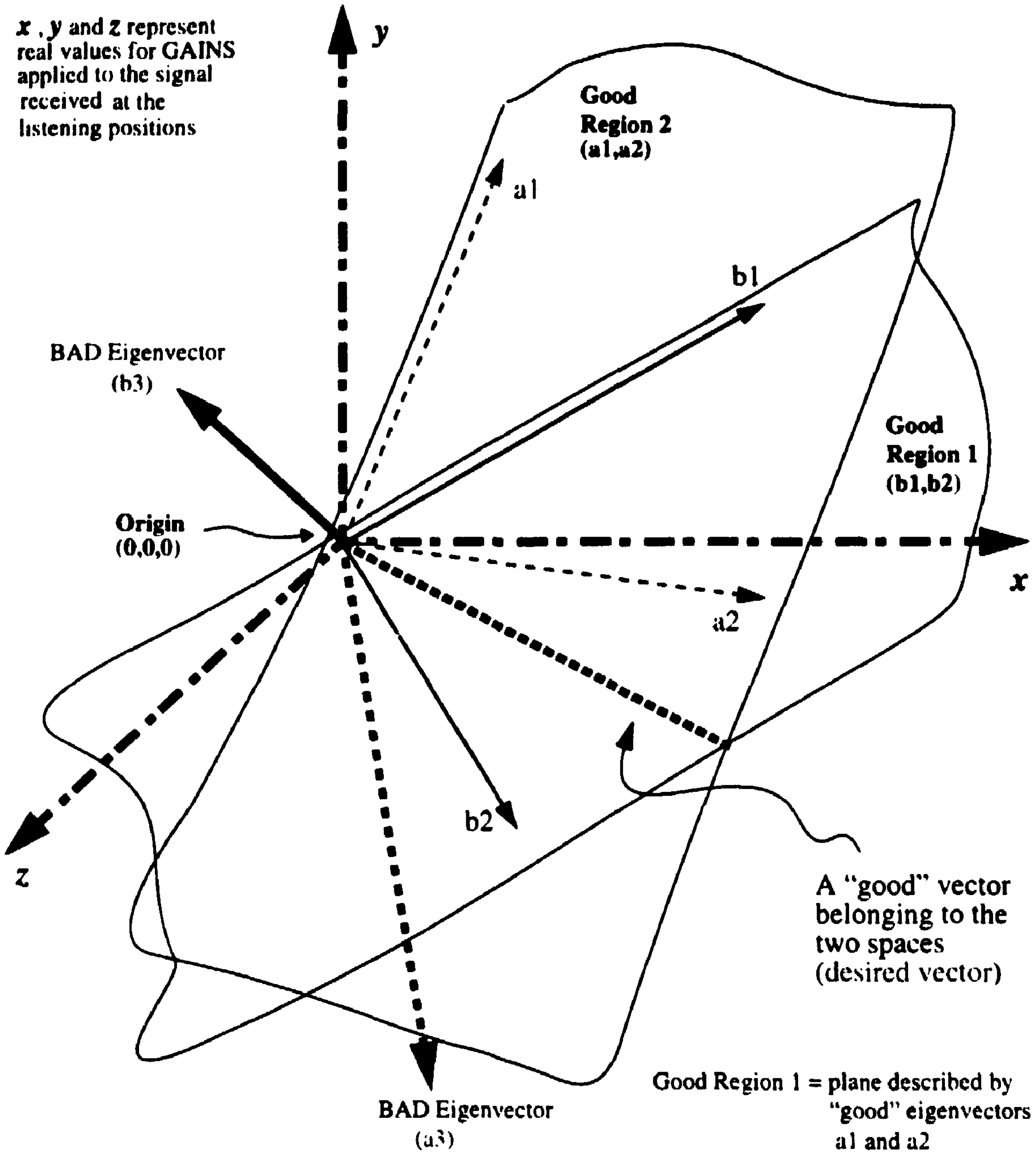
Now, a restriction on the number of frequencies to calculate the desired vector is made clearer. The number of frequencies to be used is observed to be the "number of sound sources minus one". Naturally, if one has three problematic frequencies in a 3x3 system there could be no way to satisfy all constraints, since the three planes could not generally intersect giving a vector. The only point to satisfy that condition for sure is the origin.

Reducing the example once again to a 2x2 system with two problematic frequencies, the two "good" eigenvectors (one for each problematic frequency) will intersect only at the origin. A 2x2 system with three problematic frequencies will also fall in the same category. In both cases, the designer is faced with the need to choose which of the frequencies is to be used to calculate the desired vector to be applied to the other frequencies. Another alternative is to add more degrees of freedom to the system by adding sound sources. This case has already been mentioned and would fall into the problem of inverting a non-square matrix of transfer functions to perform the equalization. This case is not investigated in this thesis.

Figure 9 shows the location of the "good" planes, the "vectors" and the intersection of the planes where the desired vector is to be placed for a 3x3 system with two problematic frequencies. The three axes x, y and z represent real valued gains measured at the listening positions. In real life, these gains are complex, and one will have phase information associated with the magnitude of each of the gains in the desired vector. The approach with complex numbers is presented in a later section.

The picture shows three eigenvectors for each of the two bad frequencies: two eigenvectors are "good" (a1 and a2 for the first frequency and b1 and b2 for the second) and one is "bad" (a3 for the first frequency and b3 for the second). The two "good" eigenvectors are the ones corresponding to what can be done with the system in terms of gains for the signal received at the listening positions. Vectors contained in each of the planes described by the two "good" eigenvectors of each frequency satisfy what is dictated by the transfer function matrix at that frequency. The intersection of the two planes indicates where the desired vector applicable to all frequencies is to be, since it satisfies the constraints imposed by both of the bad frequencies.

An analysis for the 4x4 case (four listening positions and four sources) can be extended from the 3x3 case. One more dimension can be added to the entire example. The plane



x, *y* and *z* represent real values for GAINS applied to the signal received at the listening positions

A "good" vector belonging to the two spaces (desired vector)

(a1,a2,a3)=eigenvectors of H for
problematic frequency 1

(b1,b2,b3)=eigenvectors of H for
problematic frequency 2

Good Region 1 = plane described by
"good" eigenvectors
a1 and a2

Good Region 2 = plane described by
"good" eigenvectors
b1 and b2

FIGURE 9. Spatial Analysis of Eigenvectors

generated by the best eigenvectors can now be seen as a three-dimensional space, and the two "good" three-dimensional spaces will intersect in a plane. Any vector positioned on that plane will represent what can be done with the system in terms of gains at the listening positions. Note however that for the 4x4 case, one degree of freedom (sound source) is added as well as one more constraint (one more listening position to be equalized).

The multipoint equalization algorithm will search for a vector in a region where one will find the close • solution possible to whatever was desired in terms of gains. For an *nxn* system configuration, this region has dimension *n-2* and is the intersection of the *n-1* spaces generated by the good eigenvectors of each problematic frequency.

Figure 9 represents an analysis of the magnitude of the eigenvectors and of the desired set of gains. As mentioned before, the eigenvectors have complex valued elements, and therefore an analysis for the phase should also be done. For the phase analysis, differences are introduced due to the delay from speakers to listening positions.

If the "good regions" are very close to each other, or if the eigenvectors defining the plane are very close in value, one can roughly assume that the intersection between those planes is no longer a line (or a vector) but the plane itself, and any of the vectors positioned in that plane will provide an acceptable set of magnitudes for the desired vector.

By observing the plot of the condition number over frequency in figure 7, one can immediately evaluate by the number of peaks and their position in frequency, if the equalization will be not only possible but also how complicated it will be. The more peaks, the more complicated it is to find a common "good" region among the spaces generated by the data related to the frequencies where the peaks are located.

Numerical Example: Two Problematic Frequencies

Let us now consider a numerical example employing two frequencies with bad condition numbers for a 3x3 system. The algorithm presented so far will be applied to these, in order to find a common desired vector satisfying the two constraints, and the respective new inverses of the transfer function matrices. A simple verification is done showing the same set of gains at both frequencies, demonstrating the inversion achieved.

Suppose that after analyzing the condition number of matrix H for all frequencies, two frequencies have presented ill-conditioned transfer function matrices. After extracting the eigenvalues and eigenvectors for those two matrices, correcting the eigenvector matrix and further orthogonalizing the eigenvectors for each of the two frequencies, the following results are obtained (in sequence):

(1) Two ill-conditioned matrices H, for 2 frequencies

```
H =                                 HB =
2.0000 4.0000 1.0000                2.0000 4.0000 5.0000
4.0000 7.0000 3.0000                -3.0000 2.0000 3.9000
4.0000 8.0000 1.9000                5.0000 1.9000 1.0000
```

(2) Calculating the Condition Number

```
Con =                               ConB =
762.3304                            121.8217
```

(3)Eigenvectors - matrix V, and (4)Eigenvalues - matrix D:

```
V =                                 VB =
-0.334   0.8745 0.3625              -0.1856 0.7454 -0.0823
-0.6689 0.3592 -0.4776              0.8031 0.0720 -0.7470
-0.6638 0.3260 0.8003               -0.5662 0.6627 0.6597
D =                                 DB =
11.9783 0.0000 0.0000               -0.0561 0.0000 0.0000
0.0000 -0.0157 0.0000               0.0000 6.8312 0.0000
0.0000 0.0000 -1.0626               0.0000 0.0000 -1.7752
```

(5) The corrected Eigenvector matrix (Vcorr)...

```
vcorr =                             vcorrB =
-0.3347 0.3625 -0.8745              0.7454 -0.0823 -0.1856
-0.6689 -0.4776 0.3592              0.0720 -0.7470 0.8031
-0.6638 0.8003 0.3260               0.6627 0.6597 -0.5662
```

(6) The orthogonalized matrix

```
Vort =                              VortB =
-0.3347 0.2662 -0.9039              0.7454 -0.3405 -0.5730
-0 6689 -0.7428 0.0289              0.0720 -0.8135 0.5771
-0.6638 0.6143 0.4267               0.6627 0.4715 0.5819
```

It is necessary that the correction be done for the two frequencies simultaneously. For this case, the desired vector is then found on the intersection of the planes generated by the two "good" vectors of V_{ort} for each problematic frequency. Since for each frequency involved, the vectors of V_{ort} are orthogonal, the intersection of the good planes defines a vector and is calculated by the cross product of the two "bad" vectors of each V_{ort}. This is shown in equation 21. The same result can be obtained by calculating each plane equation and determining the intersection.

$$d_{int} = [V_{ort}]_{bad} \times [V_{ortB}]_{bad} \qquad \text{(EQ 21)}$$

Continuing with the example, we obtain:

```
(7) The intersection of the two "good" planes
dint =
-0.2294
0.2815
-0.5051
```

Figure 10 illustrates the vectors of each V_{ort} and their positioning in space. The "good" vectors are represented in solid lines and the "bad" ones represented by the dash-dot line. The vector at the intersection is calculated from the plane equations of the two best eigenvectors for each frequency, and plotted..

For each frequency, new inverses of H are calculated and when used with the desired vector calculated (d_{int}) will emulate the direct inverse of H. To verify that, let us consider the example making use of the calculated desired and the previous results for H, V, D, HB, VB and DB.

Steps 8, 9 and 10 show the new inverses calculated and the direct inverses for each transfer function matrix H. The results are vectors "test_a" and "test_b" , obtained with

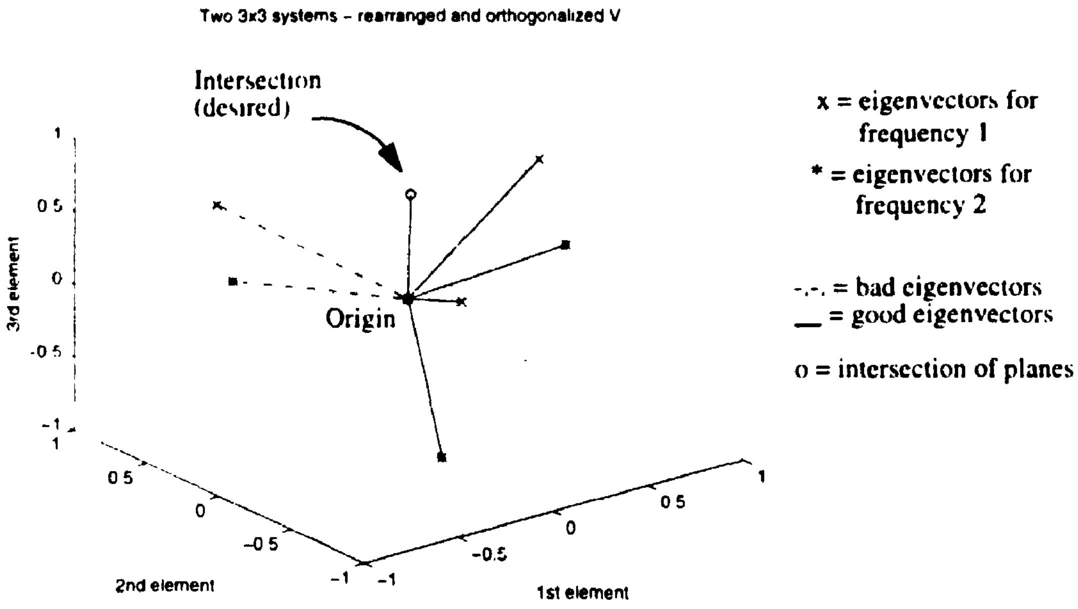


FIGURE 10. **Good and bad eigenvectors for two problematic frequencies; The intersection of good planes**

the new inverses. The results show that the same set of gains d_{int} can be used for both problematic frequencies, achieving the same result.

```
(8) Calculating new inverses for the transfer function matrix H

New_H_inv =                              New_HB_inv =
0 0278 0 2912 -0 2623                    -0.0655 0.1420 0.2229
0 0176 -0 2982 0 3757                    -1.0545 0 5608 1.1412
0 0450 0 6371 -0 5812                    0.9813 -0.4163 -0.9123

(9) These are the direct inverses

H_inv =                                  HB_inv =
-53 5000 2 0000 25 0000                  -7.9559 8.0882 8.2353
22 0000 -1.0000 -10.0000                 33.0882 -33.8235 -33 5294
20 0000 0 0000 -10.0000                  -23.0882 23.8235 23.5294

(10) Verifying . (test= H * New_inv * d)

test_a =                                 test_b =
-0.2294                                  -0.2294
0 2815                                   0.2815
-0 5051                                  -0.5051
```

The same set of gains is achieved after the equalization process, indicating the validity of the algorithm for 3x3 configurations with two problematic frequencies. Notice that the new inverses "New_H_inv" and "New_HB_inv" have much smaller gains than the direct inverses.

For Real Life: Complex Numbers

All examples thus far in the chapter have presented real valued transfer function matrices, eigenvalues and eigenvectors, inverses of H, and new inverses of H. Although real valued numbers are particularly suitable to illustrate the equalization procedure, measurements taken in a room to obtain the transfer function matrix will present - in the frequency domain - magnitude and phase. In other words, all values to be dealt with are complex values in practical situations.

Every transmission path from sound source to listening position has a delay. A constant delay should be applied somehow to the set of desired gains in order to keep phase linearity. This will accomplish a constant delay at all frequencies for each listening position. The delay to be applied is also estimated from the constraints imposed by the problematic frequencies. The matrices produced in the analysis previously done to find the vector of desired gains are also used to evaluate the phase required at the problematic frequency or frequencies.

Phase

As it has been mentioned before, in a 2x2 system, the "best" eigenvector of matrix H at a problematic frequency can be used as an indicator of what can be done with the system. This also applies for the phase. For each complex element of this best eigenvector, a phase angle is calculated. The angles at that frequency will provide the time delays that are to be kept constant for all frequencies, maintaining the phase linear through the band of interest.

The desired vector is recalculated for each frequency, in order to maintain the same magnitude for all frequencies but varying the phase according to the delay imposed by the problematic frequency. If the angle is found to be negative for any of the elements of the eigenvector, this angle is corrected in order to achieve a positive time delay. This is done according to equation 22.

$$(2\pi n + \phi_1) = \omega_1\tau_1$$

$$(2\pi n + \phi_2) = \omega_1\tau_2 \qquad \text{(EQ 22)}$$

Where ϕ_1 and ϕ_2 are the angles of the best eigenvector at the problematic frequency. We are looking for the smallest n that gives us a positive τ.

Equation 23 shows the new representation of the desired vector.

$$\dot{d}(\omega) = \begin{bmatrix} d_1 e^{-j\omega\tau_1} \\ d_2 e^{-j\omega\tau_2} \\ \dots \\ d_n e^{-j\omega\tau_n} \end{bmatrix} \qquad \text{(EQ 23)}$$

In the case of a 3x3 system, one can use - if needed - two problematic frequencies imposing constraints for the equalization. From the worst eigenvector (of matrix *Vort*) at each frequency, the constraints for the phase angle (and time delay) need to be calculated, in order for one to know what cannot be done with the system for each frequency independently. The solution is then found by satisfying the constraints imposed by both frequencies, and applied to the other frequencies of the spectrum.

Although the desired vector calculated (as shown before for a 3x3 system) provides the proper magnitude to be applied for all frequencies, the same does not apply to the phase. The angles taken from each complex component of the desired vector might not indicate an angle corresponding to what can be done - in terms of phase - with both bad frequencies. A calculation of the time delay for a 3x3 system based on the angles of the desired vector may not be possible at the problematic frequencies. The analysis based on the angles that "cannot be used" could be applied for all system configurations larger than 2x2. This, however, is left for future work.

Summary of the Equalization process

Consider the system described in the figure 11.

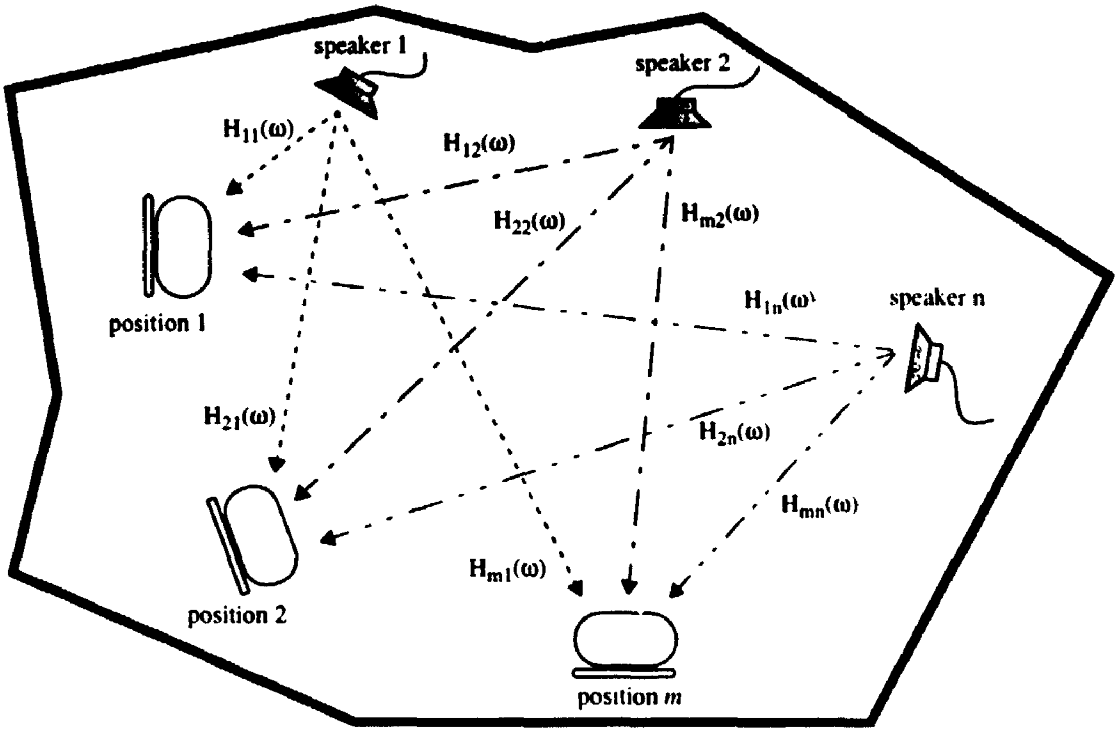


FIGURE 11. Acoustic System to be Equalized

The equalization process can be summarized in the following steps:

1) Create matrix H for the system, for n sound sources and m listening positions according to equation 24. This is done for every discrete frequency. For all cases H must be a square matrix, therefore m=n.

2) Analyze the condition number for matrix H at each frequency within the band of interest. At this point, matrix H has already been decomposed into matrix V of eigenvectors and matrix D of eigenvalues, for each frequency. Peaks in the condition number versus frequency plot will indicate ill-conditioned (or nearly singular) matrices H.

$$\begin{array}{rl}
 & \begin{array}{ccccc} \text{speaker 1} & & \text{speaker 2} & & \text{speaker n} \\ \downarrow & & \downarrow & & \downarrow \end{array} \\
\begin{array}{r} \text{position 1} \rightarrow \\ H(\omega_n) \; = \\ \\ \text{position m} \rightarrow \end{array} & \begin{bmatrix} H_{11}(\omega_n) & H_{12}(\omega_n) & \ldots & H_{1n}(\omega_n) \\ H_{21}(\omega_n) & H_{22}(\omega_n) & \ldots & H_{2n}(\omega_n) \\ \ldots & \ldots & & \ldots \\ H_{m1}(\omega_n) & H_{m2}(\omega_n) & \ldots & H_{mn}(\omega_n) \end{bmatrix}_{mics \times sources}
\end{array} \qquad \text{(EQ 24)}$$

3) For each ill-conditioned matrix H at the selected frequencies (maximum number of frequencies to be selected is "degrees of freedom minus one"), rearrange the structure of the eigenvector matrix V. The rearrangement is done in order to place the eigenvector associated with the biggest eigenvalue in the first column of the new matrix V_{corr}, and the eigenvector associated with the smallest eigenvalue (the problematic one) in the last column of that matrix.

4) Orthogonalize the corrected matrix using the Gram-Schmidt orthogonalization procedure, to orthogonalize the columns of that matrix according to the first (best) vector. A new version of the eigenvector matrix is then found, called V_{ort}.

5) Find the desired set of complex gains. For a 2x2 system, since only one problematic frequency can be chosen for reference, the desired vector turns out to be the best eigenvector of matrix H for that frequency. For systems with bigger configuration (from 3x3 up) one is looking for a vector positioned in intersection of the regions delimited by the best vectors of matrix V_{ort} for each of the problematic frequencies.

6) Calculate a new inverse of the transfer function matrix. For bad frequencies: according to equation 19, and for the other frequencies, according to equation 20.

7) Correct the phase of the vector of complex gains.

Some Important Aspects

This scheme for equalization of room transfer function at multiple listening positions avoids - at the positions involved - the problem mentioned by [Neely79] [Mourj85] and [Elliott89]: equalization of one region will cause degradation at another. For each listening position, a flat overall gain is applied to all frequencies. The gain perceived at each position might not be the same, but at each position the overall frequency response is ensured to be flat. Degradation can occur, however, in other listening positions in the venue.

By allowing different input signal configurations to the equalization filters, and how the desired gains are applied to the output signal, one could achieve source selection, stereo equalization, compensate for gains and losses in separate channels (say front and rear of a car), etc. Another case worth investigating is the case of a desired matrix instead of a desired vector. That could be applied to stereo equalization, for instance. The field is open for further development of this new equalization technique.

Conclusion

In this chapter, the algorithm for multipoint room equalization was presented and discussed. Numerical examples were given for each step of the process and a geometrical view of the problem was also presented.

The two major contributions of this technique are:

1) It shifts the problem of inverting the transfer function of a room in more than one position from finding an inverse for each of the transfer functions to finding an inverse of a matrix of transfer functions. This matrix will still be nearly singular at some frequencies, and the algorithm identifies and deals with the problematic inverse.

2) It ensures simultaneous equalization at several positions, without degeneration at any of the positions involved. The only constraint for the number of positions to be equalized is the number of "degrees of freedom" or sound sources used in the equalization process. The algorithm presented deals with square transfer function matrices only.

The next chapter will present all results obtained with experimental data and will illustrate the equalization process for 2x2 systems. Those results will be the experimental support for the algorithm just presented.

CHAPTER 4

Testing the Algorithm with Real Data

Motivation

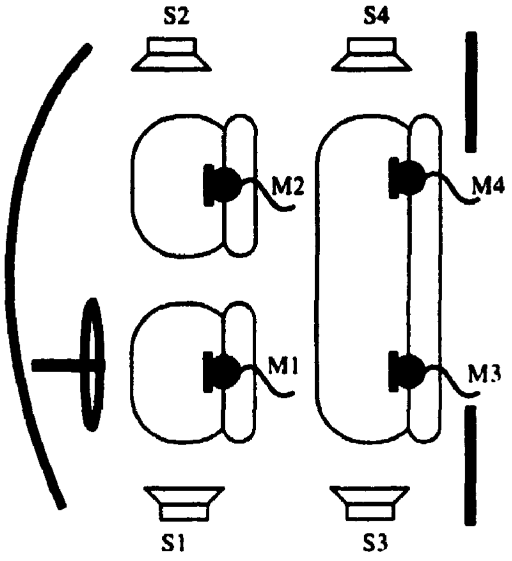
The equalization algorithm described in chapter 3 is tested using real data. This will show how efficient the approach is and provide hints for the implementation of a real-time system performing multipoint equalization in for two fixed positions, proposed in the next chapter.

Data Acquisition and Processing

The set of acoustical data in a car was provided by Ford. The impulse responses were measured with the MLSSA system [Rife89][Rife93], and all the processing done with Matlab [Matlab] by programming the algorithm previously described for the equalization of multiple listening positions in a venue.

The set of data consists of 16 files of 8192 samples, each representing the impulse response of the interior of a car in a total of 16 transmission paths, or 4 listening positions relative to 4 sound sources. The impulse responses were measured at a sampling rate of 4KHz. This set of data provides information on signals up to 2KHz, after a Fast Fourier Transformation is performed on each of the impulse responses. The complete set of impulse responses and frequency responses is presented in Appendix A.

The main results for the equalization of initially two and later three positions in the venue in which the data was collected, are presented here in detail. Figure 12 shows a diagram of one possible arrangement of loudspeakers and listening positions in a car. By observing the impulse response plots, one can evaluate the distances between sound sources and listening positions from the initial delay present in the response.



Matrix H is 2x2
(positions equalized by the nearest sources)

First Group: positions 1 and 2
Second Group: positions 1 and 3
Third Group: positions 1 and 4
Fourth Group: positions 2 and 3
Fifth Group: positions 2 and 4
Sixth Group:positions 3 and 4

Matrix H is 2x2
(positions equalized by distant sources)

First Group: positions 1 and 2 equalized by sources 3 and 4
Second Group: positions 3 and 4 equalized by sources 1 and 2

FIGURE 12. Estimated placement of sound sources and listening positions

The main analysis is done for 2x2 systems, that is a system with matrix H composed with transfer functions from two loudspeakers used to equalize two listening positions. For this configuration, this chapter will present the equalization of positions 1 and 2 using sound sources 1 and 2 only and in a second experiment, with sound sources 3 and 4 only.

The results of the equaliza'ion for other combinations of sound sources and listening positions are shown in Appendix B. The purpose of this chapter is to evaluate how well the algorithm works to the equalization of a certain range of frequencies using data collected in a real venue.

Preparing and Processing the Data

Transfer Functions and Matrix H

The Fast Fourier Transformation of the impulse responses collected will provide a set of complex data representing the transfer functions of each path, containing the magnitude and phase information for a number of "bins" of frequency. The number of discrete frequency samples was chosen to be 2048, since this would provide a good resolution in frequency without excessive computational overhead. The data obtained after the Fourier transformation is used by the equalization algorithm.

Matrix H, as defined above, is a matrix representation of the transfer functions of the venue, with its rows representing the various listening positions (in an experimental set these will be represented by microphones) and columns representing the various loudspeakers used in a setting. The composition of matrix H was described in chapters 2 and 3. Figure 13 shows an example for the equalization of positions 1 and 2 using sound sources 1 and 2.

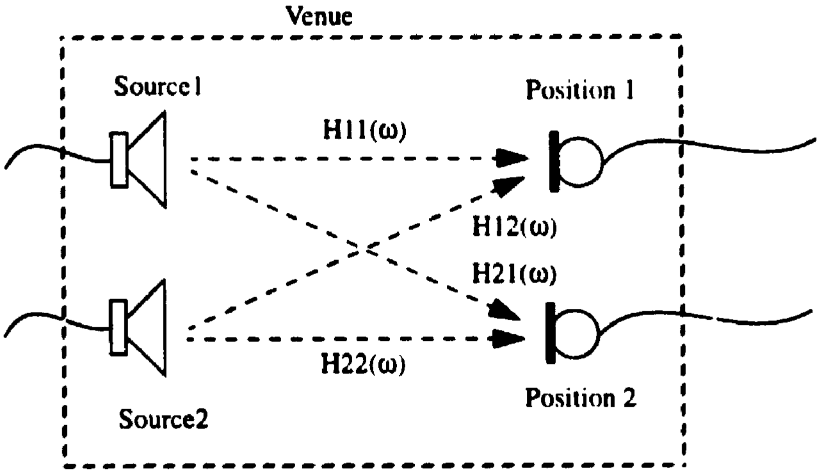


FIGURE 13. Sources 1 and 2, used for equalizing positions 1 and 2

A larger number of points used to extract the FFT will allow for better frequency resolution, as more information on the frequency contents is obtained from the time domain data. If the number of points is chosen to be too small, some relevant frequency information may not be available in the resulting data. If chosen too big, greater amount of computation which would not necessarily be crucial for the analysis would be required. Although for this experimentation this detail did not really affect the procedure except for the time spent to execute the algorithm, it was found very important in the design of a real-time implementation.

For instance, if one is interested in making use of FFT-based FIR filters for the real-time implementation, a bigger number of FFT points in the initial transformation (and a better resolution in frequency) will result in bigger transfer functions for the inverse filters calculated in the multipoint equalization algorithm. This would necessarily imply bigger buffer sizes to store not only the transfer functions themselves in memory but also "temporary" data during the computation. These buffers could grow significantly, to a point where there is a trade-off between resolution in frequency and memory availability.

More details on FFT-based FIR filters will be presented in Chapter 5, where a real-time implementation for the equalizer is proposed. For the present discussion, the computation of the FFT is done with the sole purpose of getting the frequency response of the room to perform the equalization algorithm.

For the cases to be discussed throughout this chapter and in the appendices, the frequencies analyzed will reach only a maximum of 2KHz. All results will be shown for the band of interest between 50Hz and 500Hz.

Condition Number and Eigendecomposition

As discussed, the condition number is calculated for matrix H at each discrete frequency. The most important step is to locate the frequencies of the biggest peaks in the condition number, for those peaks will indicate the frequencies where matrix H will be nearly singular. For every set of results to be presented, a plot of the condition number versus frequency is first shown, in order for the reader to visually locate the problematic frequencies and see the improvement in the results at those frequencies.

From the description of the algorithm given in the previous chapter, the whole analysis was shown to be based on the role played by the eigenvalues and the associated eigenvectors of the transfer function matrix H for the problematic frequencies. Another step in the algorithm is to find a vector of "desired" gains (corresponding to what one "would like" to have in the output in terms of gains), in order to correct the entire band of interest to have a flat - or close to flat - response. This vector will contain complex elements defining magnitude and phase of the signal at the listening positions.

Then, a new version of the inverse of H for that bad frequency is calculated from both the original matrix V and a version of the inverse of matrix D that has the "bad" element corrected. Finally, the desired set of gains is used to calculate new versions of the "direct" inverses of H at all the other frequencies, so that the new inverse, when multiplied by the set of gains, will emulate the direct inverse at all frequencies.

Analysis of Experimental Results

This section will present the best results obtained for a 2x2 system. First, the two sound sources closest to the listening positions to be equalized were used; then the two sound sources that were most distant from the listening positions were used.

This second attempt was made in order to evaluate, for instance, how well sound sources positioned in the back of the car would perform the equalization for the two front positions.

First case: two listening positions equalized by their closest sound sources

The first graph presented is the condition number versus frequency graph, since it shows the frequency where a difficult inverse of matrix H is found. This frequency will be used as a reference from where information about the system will be extracted.

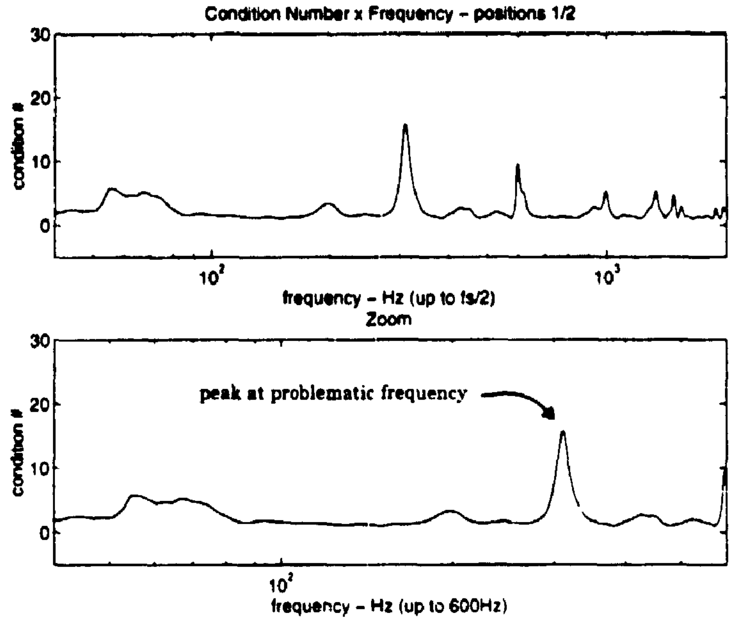


FIGURE 14. **Condition Number over frequency for positions 1 and 2**

In the figure above, between 50 and 80Hz a group of frequencies presents higher condition numbers, and one peak is clearly noticeable at around 300Hz. Since for a 2x2 system one is restricted to use a single frequency to constrain equalization, the biggest peak is used. In this case, the peak is not part of a "cluster". It stands out by itself. In all the attempts made for the different configurations, the peaks that were not clustered with others presented better results. Peaks taken from clusters often presented some undesired feature in either the overall frequency response or in the phase response for one or both the positions being equalized.

Analyzing the transfer function matrix for the frequency where the peak occurs, and executing the algorithm step by step, the calculation will provide the set of complex gains to be applied to all frequencies and the new inverse of matrix H for the problematic frequency. Following are the results

```
System composed by sources 1 and 2 and positions 1 and 2
-----------------------------------------------------------
(1) There is a condition number peak at bin # ..
n =
159

(2) Its condition number (absolute value) is ..
condition_number =
15.6494

(3) The bin corresponds to frequency (Hz)
frequency =
310 5469

(4) Matrix H for this bin is...
HH =
-0.3791 + 0.9999i 0 2514 - 1 0641i
0 4621 - 1.2934i -0.2042 + 1.0710i

(5)Eigenvectors - matrix V, and (6)Eigenvalues - matrix D
V =
0.6629 - 0 0000i 0 6676 - 0 0368i
-0 7485 + 0.0158i 0.7414 + 0.0575i
D =
-0.6375 + 2.2074i 0
0 0 0542 - 0 1364i

(7) The corrected Eigenvector matrix (Vcorr) .
vcorr =
0.6629 - 0.0000i 0.6676 - 0 0368i
-0 7485 + 0 0158i 0.7414 + 0 0575i

(8) The orthogonalized matrix
Vort =
0 6629 - 0 0000i 0 7485 + 0 0158i
-0 7485 + 0.0158i 0 6629

(9) Separating good and bad from orthogonalized matrix
good =
0.6629 - 0 0000i
-0.7485 + 0.0158i
bad =
0 7485 + 0.0158i
0.6629

(10) This is the desired one would like to have
d =
0.6629 - 0 0000i
-0.7485 + 0 0158i
```

```
(11) This is the desired that the system can give

(a) transformation matrix
tran =
1 0
0 0

(b) Desired vector of complex gains
Ddes =
0 6629 - 0 0000i
-0 7485 + 0 0158i

(12) Calculation of the time delays for the bad frequency

(a) Absolute values come from Ddes
Mod =
0.6629
0 7487

(b) Phase info from the best vector in Vort
Ang3 =
-0 0000
3 1204

(angles corrected)
Ang3 =
6 2832
3 1204

Delays allowed for Ddes
tau1 =
0 0032
tau2 =
0 0016

(13) Correction for the inverse of the eigenvalue matrix D
tran1 =
-0 1208 - 0 4181i 0
0 0

(14) New Inverse of H for problematic frequency
PsH =
-0.0438 - 0 2121i 0 0642 + 0 1838i
0 0546 + 0 2384i -0 0769 - 0 2060i

(a) absolute values in dB
PsH_abs =
-13 2869 -14 2114
-12.2304 -13 1550

Compare  absolute values (dB) of the direct inverse of H
Direct_inverse =
10 1898 10 2142
12 1952 10 0214
```

FIGURE 15. Example of the equalization process in 14 steps

Applying the results to all frequencies, one finds the values of the new inverse of matrix H. The next graphs show these values. The new inverse of matrix H is plotted together with the "direct" inverse of H, in order for the improvements achieved to be more noticeable. The plots are organized element by element, for the 2x2 matrices (both representing inverses), and present both magnitude and phase of the elements versus frequency.

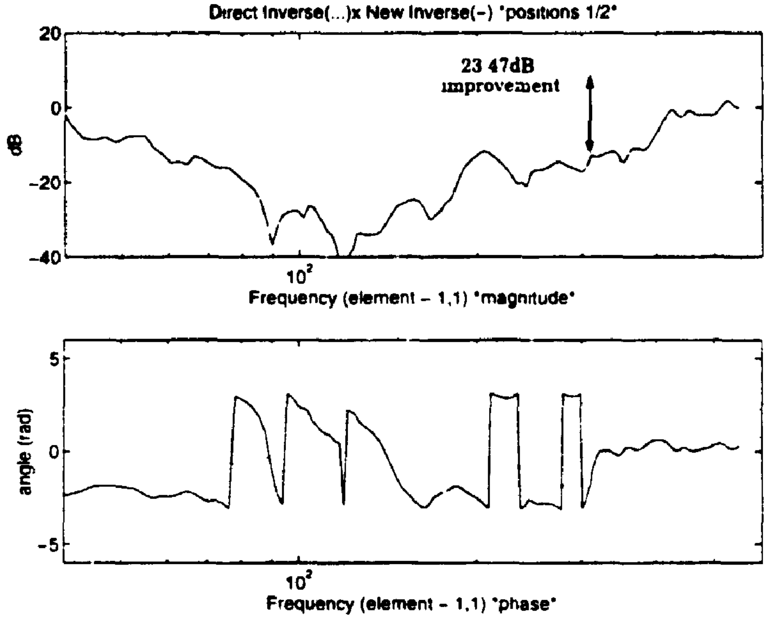


FIGURE 16. Element 1,1 of the Direct Inverse of H (dotted) and the New Inverse of H (solid) - magnitude and phase

The first portion of figure 16 shows already the improvement achieved with the new inverse found for matrix H. One of the goals of the equalization algorithm is to reduce the peak in the direct inverse caused by a nearly singular transfer function matrix H at a particular frequency. The frequency where this occurs has been identified in figure 15, and the expected peak on the inverse of the transfer function is shown in magnitude plot in figure 16. The improvement achieved is over 23 dB at the problematic frequency.

The same can be perceived for the second element of the direct inverse and the new inverse of matrix H. This is shown in figure 17.

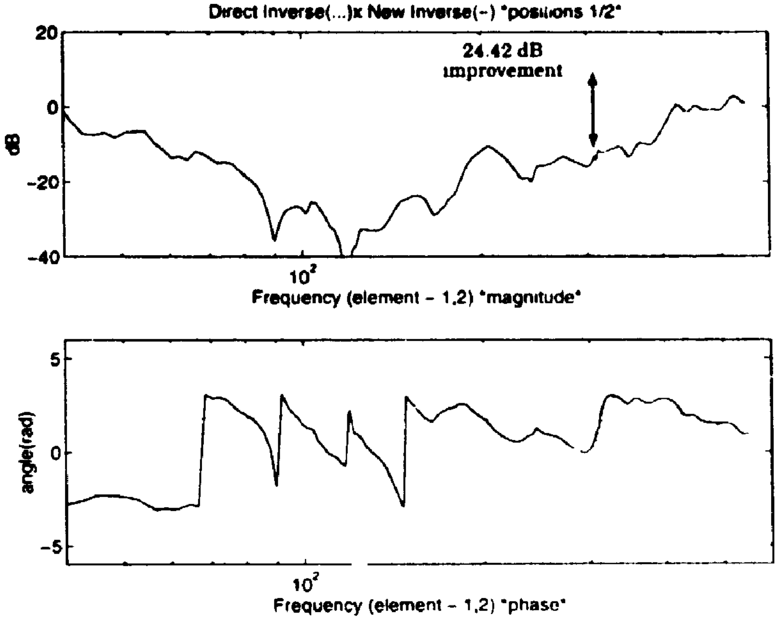


FIGURE 17. Element 1,2 of the Direct Inverse of H (dotted) and the New Inverse of H (solid) - magnitude and phase

Figures 18 and 19 show the elements of the second row of the direct inverse and the new inverse of the transfer function matrix versus frequency. Both figures show improvement for the new inverse of the transfer function matrix, since high gains in the inverse are avoided. For the particular frequency where the peak in the direct inverse is located, the results show that the peak is definitely avoided for this particular 2x2 configuration.

The leading phases shown in the next figures are not a concern at this point of the analysis, since the new inverse is always used in the equalization algorithm in conjunction with the desired set of complex gains previously calculated; we actually implemented (H_{NIH}^{-1} . d), not H_{NIH}^{-1} alone. The desired set of gains will dictate the phase delay to be given to all frequencies, based on the one imposed by the problematic frequency. The phase problem is therefore approached in the process of choosing the time delay to be applied by the desired set of gains at each frequency, and that result will be clear in figures 20 and 21.

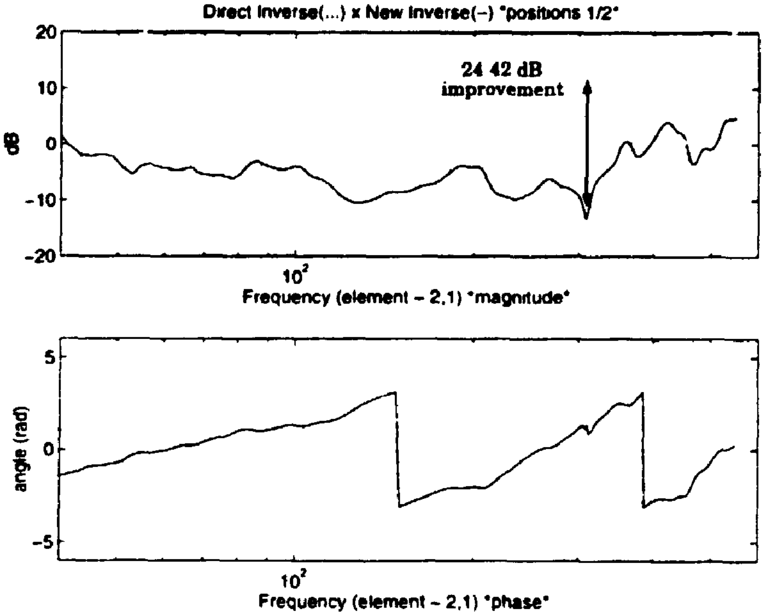


FIGURE 18. Element 2,1 of the Direct Inverse of H (dotted) and the New Inverse of H (solid) - magnitude and phase

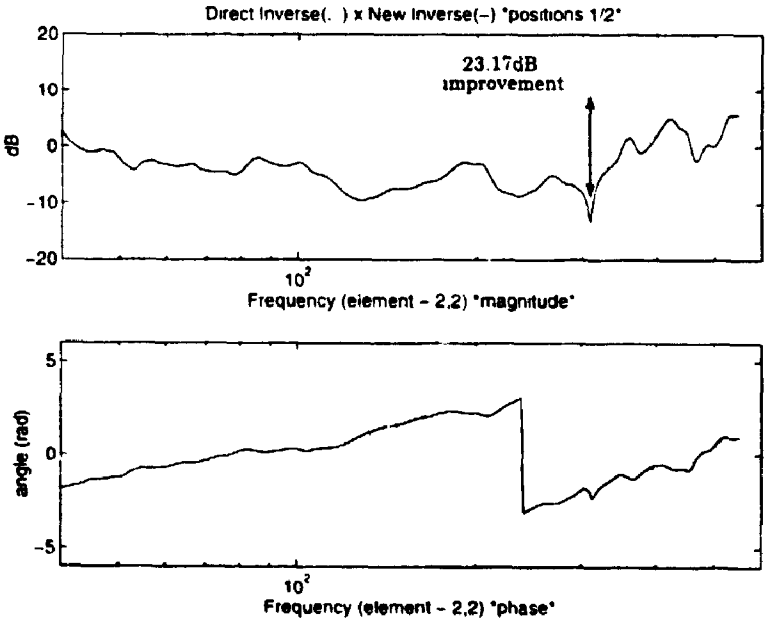


FIGURE 19. Element 2,2 of the Direct Inverse of H (dotted) and the New Inverse of H (solid) - magnitude and phase

The results of the equalization procedure for a system composed by 2 loudspeakers used to equalize two listening positions are then presented below.

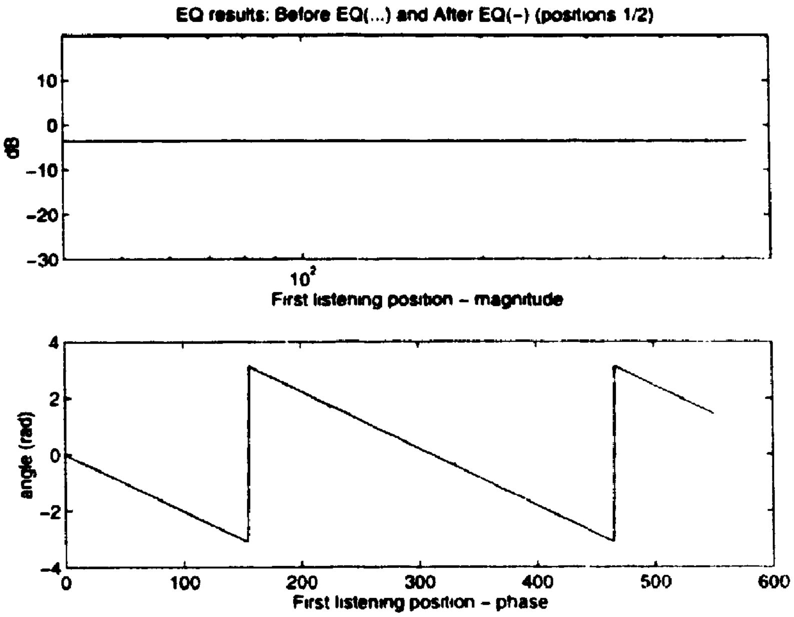


FIGURE 20. Results of the equalization for first listening position - magnitude and phase (dotted line is before equalization)

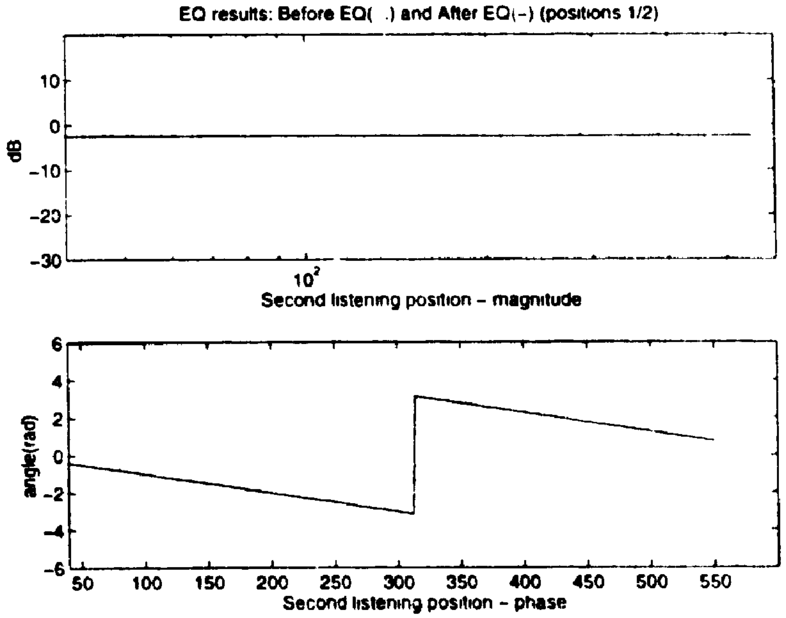


FIGURE 21. Results of the equalization for second listening position - magnitude and phase (dotted line is before equalization)

Figures 20 and 21 show the result of the equalization process for frequencies between 50 and 500Hz. The magnitude plot presents a flat spectrum for the frequencies of the band of interest, and from position 1 to position 2 a small difference in gain is noticed (around 1dB), as expected, since the absolute value of the elements of the desired vector present the same difference.

As for the phase, a time constant has been introduced to the elements of the desired vector in order to correct the phase delay of all frequencies according to the phase required at the problematic frequency.

In this set of results, improvements were observed for the new inverse of the transfer function matrix H in comparison to the direct inverse, the expected flatness of the resulting transfer function at each listening position was achieved, and the phase was corrected in order to allow for a linear phase response for the room at the particular listening positions. Equalization was achieved for two positions in a venue, and the equalization of the first did not cause degradation of the second.

Second Case: two listening positions equalized by distant sound sources

This second case was investigated using the sound sources that were positioned farther from the listening positions than the other two sources used in the previous case.

Observing initially the condition number over frequency graph presented in figure 22, a peak is noticed between 50 and 60 Hz. This peak indicates the frequency used as reference for the equalization process.

After applying the equalization algorithm to the data for this case, the improvements observed for the new inverse of the transfer function matrix H were somewhat better than the ones presented in the previous case, particularly at the problematic frequency. The

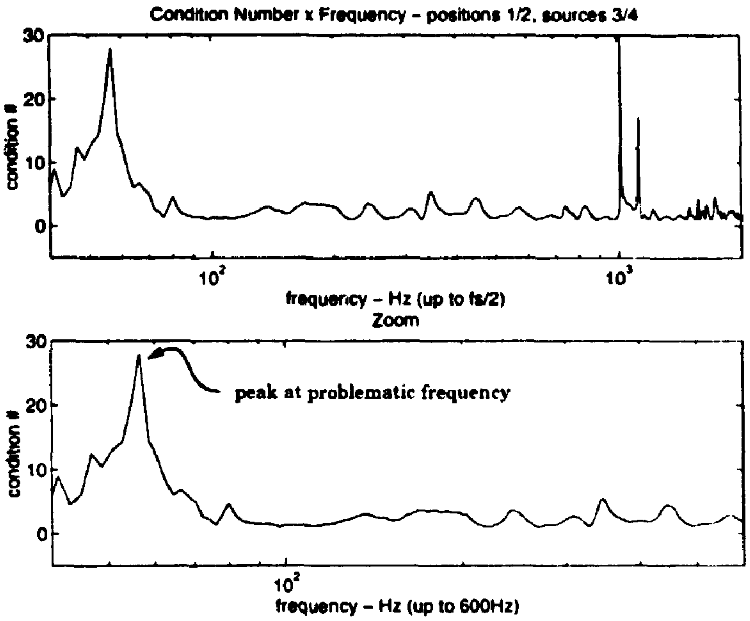


FIGURE 22. Condition Number over frequency for positions 1 and 2 using sound sources 3 and 4

objective of reducing the peaks present for each element of the inverse matrix at the problematic frequency was achieved. The improvements in dB are shown in the graphs.

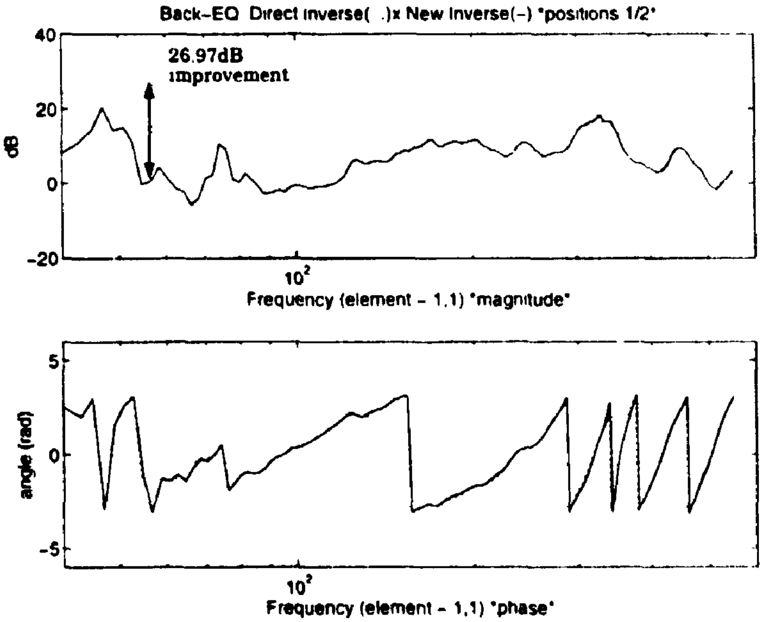


FIGURE 23. Element 1,1 of the Direct Inverse of H (dotted) and the New Inverse of H (solid) - magnitude and phase

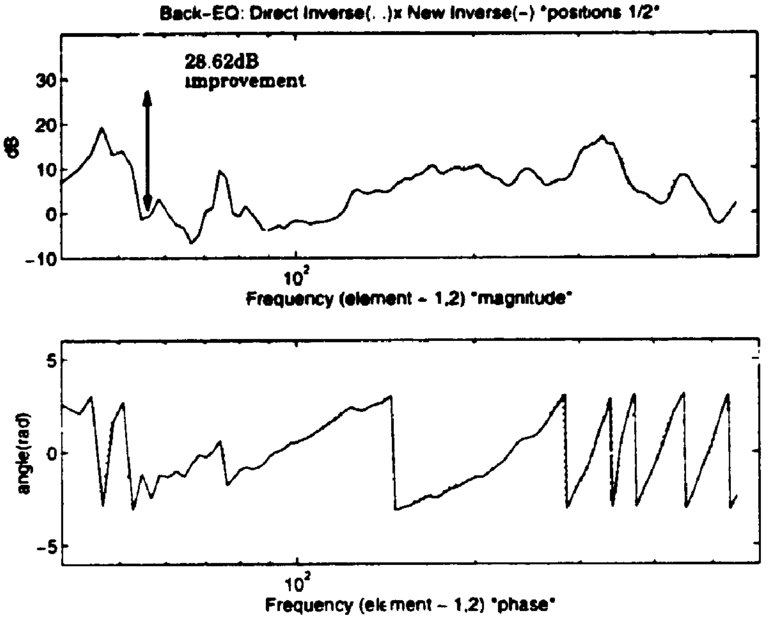


FIGURE 24. **Element 2,2 of the Direct Inverse of H (dotted) and the New Inverse of H (solid) - magnitude and phase**

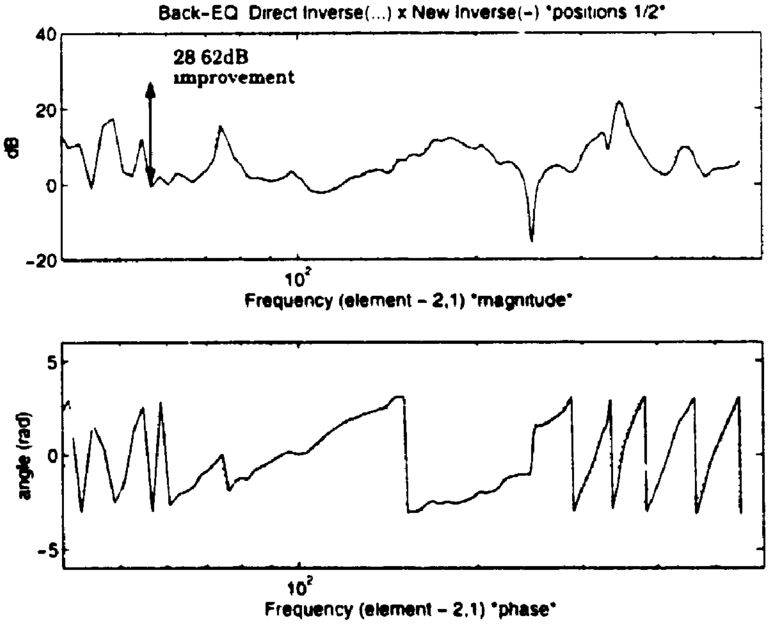


FIGURE 25. **Element 2,1 of the Direct Inverse of H (dotted) and the New Inverse of H (solid) - magnitude and phase**

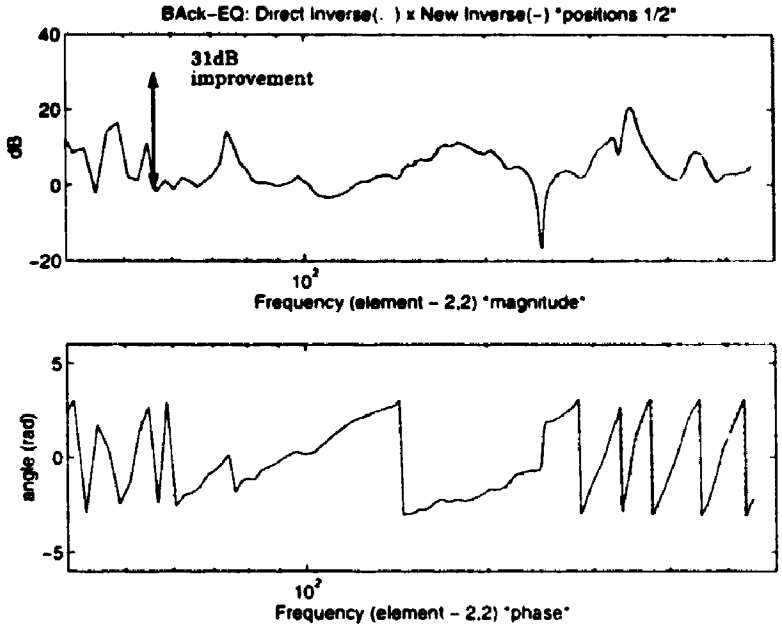


FIGURE 26. Element 2,2 of the Direct Inverse of H (dotted) and the New Inverse of H (solid) - magnitude and phase

Following are the results obtained after the equalization at each listening position.

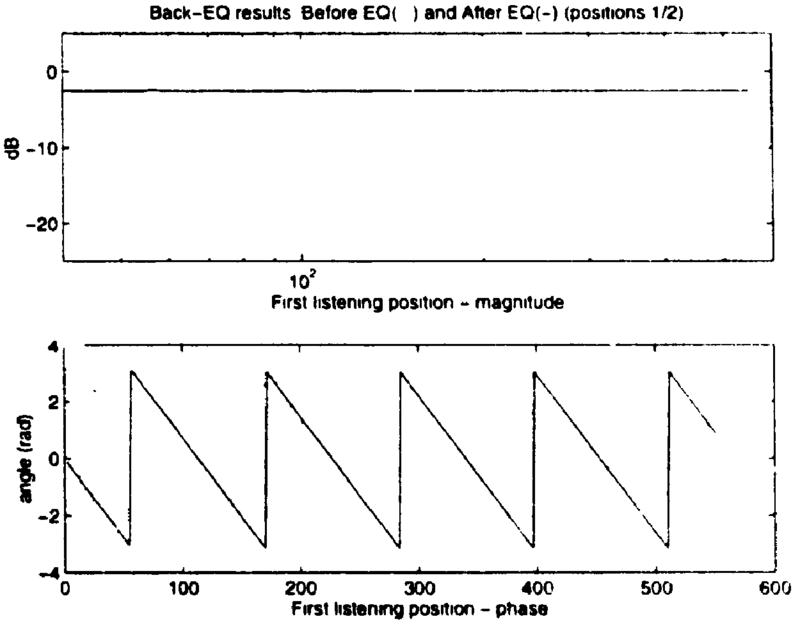


FIGURE 27. Results of the equalization for first listening position - magnitude and phase (dotted line is before equalization)

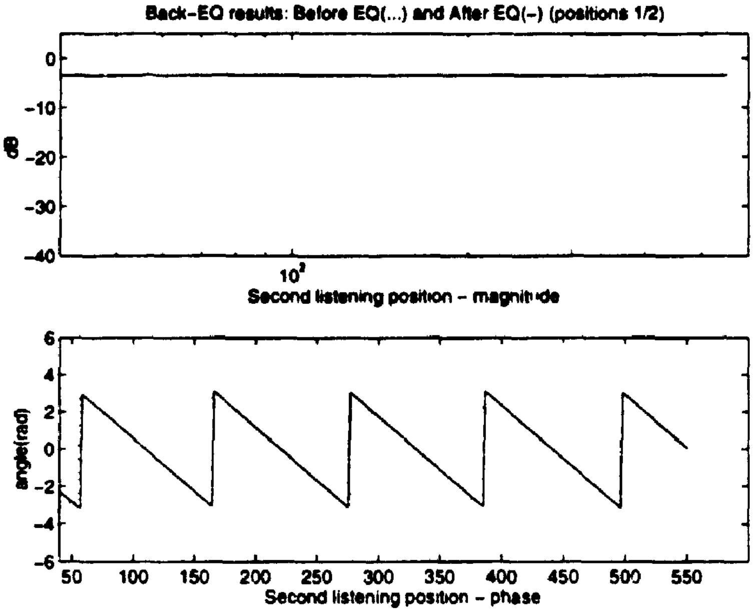


FIGURE 28. **Results of the equalization for second listening position - magnitude and phase (dotted line is before equalization)**

Similar to the previous case, the resulting transfer function presents a generally flat spectrum and a linear phase behaviour within the band of interest. The difference in gain (magnitude) from one position to the other was around 1.1dB. For different configurations this difference in gain is much greater, as can be seen in Appendix B, where all the results are shown.

Comments on the Overall Results

In this chapter, two sets of results were shown for 2x2 system configurations. The complete set of system configurations and their initial measurements is found in Appendix A. The results after the equalization procedure are shown in Appendix B, for each of the configurations for a 2x2 system.

The following tables list in dB the improvements found in 2x2 systems, for the new inverse of matrix H at problematic frequencies compared to the original inverse of matrix H. Each element is compared for the problematic frequency. Table 3 presents the elements of the first row of both the "direct" inverse of the transfer function matrix H and its new inverse, and table 4 presents the elements of the second row of these matrices.

TABLE 3. First rows of the direct inverse of H and the new inverse - Improvements in dB

positions with sources used (s)	**Direct Inverse of H element (1,1)**	**New Inverse of H element (1,1)**	**Direct Inverse of H element (1,2)**	**New Inverse of H element (1,2)**	**Problematic Frequency**
1 and 2 (with s1 and s2)	10.1898dB	-13.2869dB	10.2142dB	-14.2114dB	310.5469Hz
1 and 3 (with s1 and s3)	4.5738dB	-2.1287dB	12.6175dB	-16.9656dB	68.3594Hz
1 and 4 (with s1 and s4)	-5.6910 dB	-3.1727 dB	-3.8648 dB	-28.5533 dB	212.8906 Hz
2 and 3 (with s2 and s3)	13.4861 dB	0.2054 dB	25.0355 dB	-7.7582 dB	80.0781 Hz
2 and 4 (with s2 and s4)	9.5068 dB	0.8395 dB	13.0738 dB	-11.1835 dB	64 4531 Hz
3 and 4 (with s3 and s4)	9.1377 dB	-2.7154 dB	14.2329 dB	-6.5767 dB	207 0312 Hz
1 and 2 (with s3 and s4)	27.5615 dB	0.5850 dB	28.0754 dB	-0.5463 dB	56.6406 Hz
3 and 4 (with s1 and s2)	33.9662 dB	-7.1629 dB	32.6868 dB	-7.5759 dB	48.8281 Hz

TABLE 4. Second Rows of the direct inverse of H and the new inverse - Improvements in dB

positions with sources used (s)	**Direct Inverse of H element (2,1)**	**New Inverse of H element (2,1)**	**Direct Inverse of H element (2,2)**	**New Inverse of H element (2,2)**	**Problematic Frequency**
1 and 2 (with s1 and s2)	12.1952 dB	-12.2304 dB	10.0214 dB	-13.1550 dB	310.5469 Hz
1 and 3 (with s1 and s3)	14.4264 dB	-15.1566 dB	27.2885 dB	-29.9934 dB	68.3594 Hz
1 and 4 (with s1 and s4)	12.5245 dB	-12.1639 dB	21.0363 dB	-37.5445 dB	212.8906 Hz

TABLE 4. Second Rows of the direct inverse of H and the new inverse - Improvements in dB

positions with sources used (s)	Direct Inverse of H element (2,1)	New Inverse of H element (2,1)	Direct Inverse of H element (2,2)	New Inverse of H element (2,2)	Problematic Frequency
2 and 3 (with s2 and s3)	20.5219 dB	-12.2718 dB	32.9344 dB	-20.2354 dB	80.0781 Hz
2 and 4 (with s2 and s4)	17.8959 dB	-6.3614 dB	24.7266 dB	-18.3844 dB	64.4531 Hz
3 and 4 (with s3 and s4)	13.8579 dB	-6.9518 dB	18.1629 dB	-10.8132 dB	207.0312 Hz
1 and 2 (with s3 and s4)	28.1052 dB	-0.5166 dB	29.7264 dB	-1.6479 dB	56.6406 Hz
3 and 4 (with s1 and s2)	34.4368 dB	-5.8259 dB	33.0104 dB	-6.2389 dB	48.8281 Hz

In general, improvement is achieved over the entire band with the new version of the inverse of matrix H, if compared with the direct inverse. Improvement is expected and will occur at the problematic frequency, since by applying the equalization algorithm one is aiming for a version of the inverse that is free of the problematic (bad) component of the direct inverse. Only one exception was observed from the sets of data presented: element (1,1) of the direct inverse of H for the configuration using sources 1 and 4 in the equalization for positions 1 and 4 was observed to be lower than the corresponding element of the new inverse of H. The difference was calculated to be 2.51dB.

For the other frequencies of the spectrum, a new inverse was calculated so that when used with the desired set of gains it would emulate the direct inverse of H. This new version of the inverse presents in general lower magnitude than the direct inverse. For some frequencies the magnitude of the new inverse of H is higher than the one of the direct inverse of H. This does not happen at the problematic frequencies chosen - where the correction is applied - and might indicate the need for more freedom to correct other problematic frequencies. This implies that more sound sources to correct other bad features of the transfer function of a room for a particular position need to be used.

Peaks in the new inverse were observed to occur when another bad feature was present at a frequency in the transfer function of the room for a position, and that frequency was not the one used by the equalization process.

As for the equalization of one position in a venue causing degradation on the transfer function relative to another position, the algorithm presented here avoids that effect for the two listening positions involved, but degradation is expected to occur somewhere else in the venue. The quantitative analysis of this degradation is left for future work.

Conclusion

In this chapter, simulations were performed with real data in order to evaluate if the multipoint equalization algorithm presents satisfactory results. Experiments with data collected in a car showed that equalization can be achieved for the band between 50 and 500 Hz for two positions. The equalization algorithm applied to sets of real data relative to two listening positions and two sound sources produces flat overall response at each of the positions. The results also show that equalization is achieved for one position without degradation of the response at the other position involved. There is a difference in the magnitude of the signal received at each position, but the overall flatness of the spectrum - which is the main objective of the algorithm - is achieved.

The correction of the phase was also proven to be possible, by applying a constant time delay to the complex gains used in the correction of all frequencies.

With these results, the search for a real-time implementation can be initiated. This is the objective of the next chapter and also the goal of future research.

CHAPTER 5

A Real-Time Implementation

Introduction

This chapter proposes an implementation in real-time of a multipoint room equalizer on a DSP-based platform. The description to be presented applies for systems using two loudspeakers used for the equalization of two listening positions.

For the 2x2 case, the two main parts of the system to be proposed were actually implemented. These parts are the analysis/synthesis stages and the filter in the frequency domain. This test was performed in order to establish that the implementation of the algorithm is feasible using current technology.

Three main features are present in the real-time implementation: filter banks, multirate signal processing and filtering in the frequency domain. Details of these topics are included in the description of the system proposed.

Description of the Hardware

The studies made for the implementation of the multipoint equalizer point towards the use of a DSP based platform, programmed in C or assembly. We have chosen to use an evaluation kit for the Analog Devices ADSP21020 signal processor [AD93], which is a

32-bit floating point signal processor operating at 33MHz. This kit includes an in-circuit emulator and a PC or Sun-based simulator for the ADSP21020 processor together with the evaluation board itself.

The board allows for a two channel input/output with a 16bit A/D and D/A converter known as the "audioport". The A/D and D/A converter is interfaced with the ADSP21020 through a fixed point signal processor acting as a controller, the Analog Devices ADSP2111. The controller not only translates the incoming and outgoing data, but also provides the audioport with information about the sampling frequency to be used (the user can choose between a fixed set of sampling frequencies), the type of input and output, and the input and output gains. The programmer has then full control over the audioport.

This configuration seems to be a standard among manufacturers of signal processors, such as Motorola and Texas Instruments. The choice for using the Analog Devices processor was mainly due to a very user-friendly assembly language, and a simulator available for the Sun workstations. The competitive price (for floating point processors), and the availability of a library of basic DSP routines also influenced the decision.

Data Acquisition

Similarly to the data collected for the simulation, the system suggested to be used for room data collection is the MLSSA system, a commercially available system for acoustic measurements based on maximum-length sequences (MLS) [Schr79] [Bori83] [Bori85] [Rife89] [Rife92] [Dunn93]. The MLSSA system is suitable due to the simplicity of its operation and precision of its measurements.

The use of maximum-length sequences has lately become a standard technique for acoustic measurements. Among the many advantages in using MLS based systems are the

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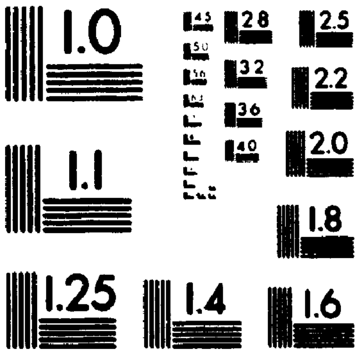

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ease of generating the sequence, the speed in computation to produce the measurements (because it is a binary sequence, it involves only additions and subtractions) and the fact that these sequences are deterministic. The latter is very useful, since it allows for the same sequence to be used in repeated measurements.

Maximum-length sequences are also known as "pseudo-random sequences" due to their similarity with white noise. The autocorrelation of a maximum-length sequence approximates a perfect impulse, and its spectrum is flat over the audible frequency band. The impulse response of a system is obtained by cross-correlating the data collected with the sequence originally sent into the system to be measured.

In order to improve the signal-to-noise ratio in the results, MLS based techniques allow for averaging the response over a number of measurements taken, more efficiently and precisely than Periodic Impulse Excitation [Preis77][Berm77] for instance. This is due to the deterministic nature of ML sequences.

Other acoustic measurement techniques like Time Delay Spectrometry [Heysr67] [Vandk86] [Grein89], will provide the user with the direct measurement of the frequency response of the room, instead of measuring the impulse response first and calculating the frequency response. This technique involves the use of matched filters [Heach93].

Implementation Details

The main purpose of the system to be implemented is to equalize the overall transfer function of a room at two positions simultaneously, for frequencies up to 500Hz. The equalization algorithm presented in the previous chapters devises a new inverse for a matrix of transfer functions, providing a "pseudo" inverse transfer function for each transmission path. The equalizer needs this information transformed into filter transfer functions.

To represent frequencies up to 500Hz without aliasing, one would need at least 1KHz of sampling frequency. Let us consider an incoming signal sampled at 8KHz. If one is interested in equalizing the lower part of the signal, there is no need to sample that band using 8KHz.

Downsampling that band at some point will save a considerable amount of computation time that could be used to perform other tasks. The upper frequency band, however, needs to be kept unchanged. This clearly indicates the need for a filter bank and multiple sampling rates.

The transfer functions of the filters used to equalize responses at two listening positions are obtained from the set of gains and the new inverse of matrix H previously calculated. We recall that all the calculation was done in the frequency domain. Filtering in the frequency domain should be considered, rather than applying an inverse Fourier transformation and use the time domain coefficients to perform the filtering on the band of interest.

Figure 29 shows a simplified block diagram of the proposed system. For illustration purposes, let us assume a sampling frequency of 8KHz.

The filter bank seen up front separates the signal into low and high frequency bands. The low frequency band is the one where the equalization will eventually take place. Since this band has already been limited, a decimator brings the sampling frequency down by a factor M, depending on the passband of the filter used. For example, if the band were split in half (2KHz = $\pi/2$) the decimation would be done by a factor of 2.

The next stage is an FFT-based FIR filter to perform the filtering in the frequency domain. Since the sampling frequency has been reduced at this stage, more computation time is

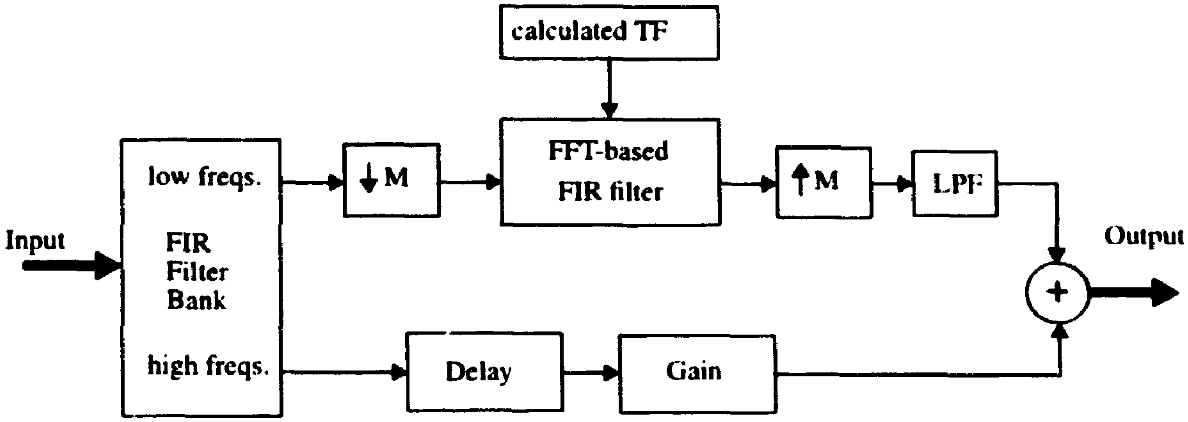


FIGURE 29. Simplified block diagram for one channel

available to perform the FFTs, multiplications and IFFTs involved in the filtering process. After the frequency domain filtering, an interpolator of the same order M as the decimator is needed to bring the sampling frequency back to its original value. This interpolation expands the frequency band by a factor of M, and also attenuates the frequencies by the same factor. A low pass filter is then used to prevent aliasing, and a gain must be given, so that the filtered signal can be added to the high frequency component of the original input.

The low pass filter used after the interpolator can have the exact same characteristics of the one used for low pass filter in the initial filter bank. In hardware implementation terms, if the same filter is used many times, there is only one location in memory (of size equal the number of taps of the filter) needed to store the set of coefficients of many filters.

The high frequency path in this case is not processed, and it is just stored in a buffer with some delay added. This delay is to compensate for the delay introduced in the low frequency portion by the FFT-based FIR filter and the low pass filter after the interpolator. After the two paths are added, the output signal is now a filtered version of the original signal, with only the lowest portion altered by the equalization filter.

Band splitting the signal once and downsampling the band of interest could require a great amount of computation power, particularly if the sampling frequency is high and the band of interest is narrow. For instance, if a bandpass filter is used to select the band of interest before the equalization, a filter of approximately 250 taps would be required if 8KHz is used as sampling frequency for the incoming signal. The solution is to perform the two tasks in more than one stage, consecutively. After the processing is done on the lowest frequency band, the results obtained at each reconstruction stage will be interpolated and added to the respective high frequency components.

By expanding this concept, it is clear that one can break the incoming signal consecutively using filter banks and decimate the output of each filter bank before it enters the next bank. One can conclude therefore, that the main motivation for using filter banks and decimators is to accomplish filtering at a low computation cost and save computation time if heavy computation is to be done at the top level (lowest sampling frequency) of the multirate system.

That turns out to have yet a third advantage for the implementation of the equalizer. If the spectrum of the incoming signal is always split in halves, one can take advantage of a particular design of FIR filter called "halfband" filter [Ifeach93]. For this design, a high pass filter can be obtained from a low pass just by changing the signs of some coefficients. One can, therefore, use the same coefficients through the entire process, employ small order filters, taking advantage of the particular filter impulse response for a computationally efficient code.

We now consider a system using a sampling rate of 16KHz, to be compatible with the standard used in high quality telephone systems. From this point on, all the descriptions are relative to the detailed system shown in figure 31. The equalizer can be divided into three main portions: the initial filter banks and downsampling (decimation) stage, the equalization stage and the upsampling and reconstruction stage. These three stages are described in detail in the next section.

Filter Banks and Downsampling

We use filter banks known as "Quadrature Mirror Filters (QMF)" to separate the target frequency band of the incoming signal. QMF filter banks have been used in many applications [Galan84][Swami86][Vaidy87][Nguyen89] and extensively studied in conjunction with multirate signal processing techniques [Vaydi93]. The proposed implementation makes use of three QMF filter banks with halfband high pass and low pass filters followed by decimators by factors of 2. One more bank is used before the FFT-based FIR filter where the equalization actually takes place.

The halfband filters mentioned previously were designed according to [Ifeach93]. These were chosen to have an odd number of coefficients (17 taps), since this allows for a design of high pass filters directly from their low pass counterparts just by inverting the sign of every other coefficient, starting from the second one [Oppenh89]. Figure 30 shows the impulse and frequency responses of the complementary low pass and high pass filters. The phase response is guaranteed to be linear, since these are FIR filters.

Another characteristic of the halfband filters is to have zeros alternating with coefficients except for the three coefficients at the central portion of the impulse response. The middle coefficient must be 0.5. This allows for an optimal implementation of the filter bank, since only non-zero coefficients will act in the computation, and therefore the filtering routine can be greatly optimized.

As shown in figure 31, after each of the QMF filter banks, a decimation by a factor of two is made. Since we have divided the incoming signal into half bands, it is now possible to

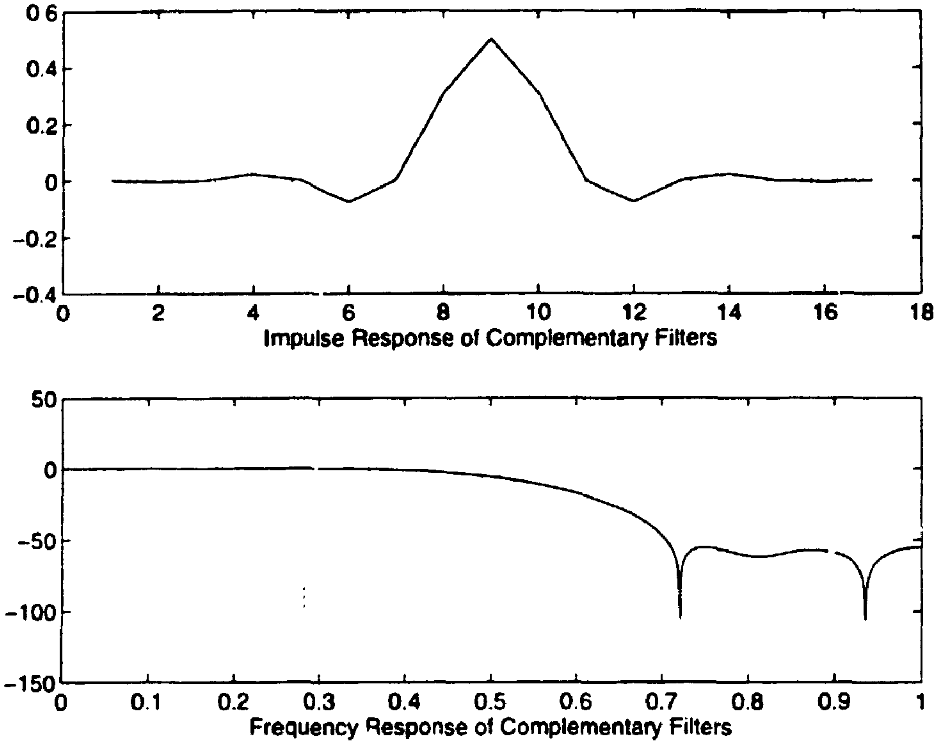


FIGURE 30. Impulse and Frequency Response of Filters in QMF Bank

keep the sampling rate of the high frequency portion of the signal and decimate (downsample) the low frequency portion by a factor of two. This is a very simple process to implement in the actual code, since only one of every two outputs of the low pass filter is actually sent to the input of the next QMF bank.

After the filter bank, the samples of the high frequency path are placed in a delayed buffer, until an output is produced by the interpolator of the corresponding stage.

Furthermore, for every sample discarded by the decimator, a zero can be inserted into the delay line of the correspondent low pass interpolation filter. In other words, further in the process, for every sample discarded, a zero will eventually be inserted between incoming samples of the interpolation filter delay line. These filters will be described further on.

The Equalization Stage

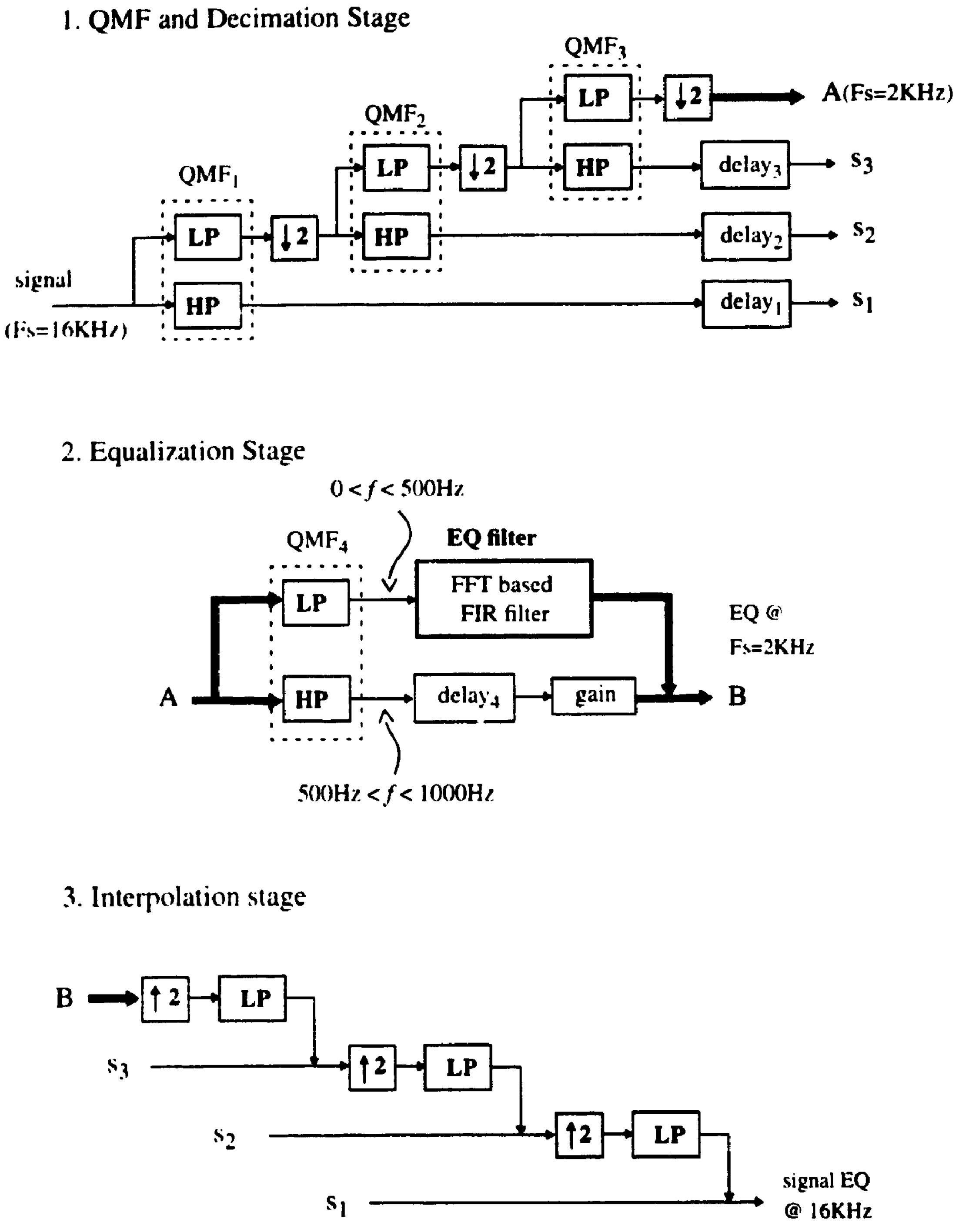
The low frequency portion of the third (top) QMF/decimation step will be then passed through a fourth QMF filter bank similar to the ones used before. This fourth filter bank will again divide the band in half and the bottom half will be fed to the FFT-based FIR filter, where the coefficients have been prepared already according to previous calculation.

The high frequency portion of the signal after the fourth filter bank will be stored in a delayed buffer, in order to match the delay imposed by the FFT-based FIR filter. Filtering is done in the frequency domain, by implementing an overlap-save scheme [Oppenh89]. This filtering scheme is particularly efficient for very long filters (see [Gard95]).

The incoming samples are collected in blocks, transformed to the frequency domain by FFT, multiplied by the transfer function of the filter and transformed back to time domain to be output. The transfer function used is obtained by the Fourier transformation of a zero-padded impulse response of the desired filter. Due to the circular nature of the FFT, zero padding is done to avoid overlapping of relevant data. The result is similar to linear convolution in the time domain. More details on frequency domain filtering can be found in [Meyer91][Gard95].

A program was written to perform filtering in the frequency domain, making use of a radix-4 FFT routine. This routine was optimized by Analog Devices [AD93] and it is out of the scope of this thesis to describe the routine itself. Different routines for the computation of FFTs are thoroughly explained in [Brigh74]. Details on the computation load of discrete Fourier transforms can be found in [Heid86].

The original filtering routine designed operates in two channels, using two different transfer functions. The incoming signal is a single signal that is sent to two paths to be



1. QMF and Decimation Stage

2. Equalization Stage

3. Interpolation stage

FIGURE 31. Detailed block diagram of the equalizer

processed (equalized) and then drive two sound sources to equalize two listening positions.

The results found by applying the equalization algorithm to the room data need to be prepared and input as filter transfer functions for the FFT-based FIR filter. The preparation of the data obtained from the equalization algorithm for the overlap-save routine implemented in the filter is left for future work.

One of the problems expected to happen after the filtering is the transition between the equalized low-frequency portion of the signal and the non-equalized high-frequency. Like in [Elliott94], there is a need of smoothing this transition region, otherwise the transition could be noticeable for the listener. This smoothing is done by choosing the proper gain to be given to the paths on the reconstruction stage.

The Reconstruction Stage

The reconstruction stage is constituted by interpolators, low pass filters and adders. The low pass filters used have the same coefficients as the one in the QMF filter banks. Moreover, the routine to perform the filter can be greatly optimized, very similarly to what was done for the QMF bank. In Appendix C we show the QMF routine and the low pass interpolation filter routine. Both routines increase the filtering speed almost twofold. Since the filters used were not high order (greater than 30, for instance) the routines were not implemented using "loops". This was done to make the debugging of the code easier during tests.

The main characteristic of this implementation is that for every sample discarded by the decimation process done in the first stage, a zero is placed in the delay line of the corresponding interpolation filter. In other words, when the decimation is done to the highest band (first QMF filter bank), for every sample discarded in the decimator, one zero is added into the delay line of the corresponding low pass interpolation filter. This is made possible since the interpolation is done also by a factor of two.

Aligning the Delays

The delay present in the high frequency path is always dictated by the band immediately above (lower in frequency). In order to estimate the delay of a high frequency path, one should first estimate the delay in the lowest frequency stage (in this case the equalization filter) and then gradually estimate the higher frequency paths.

The easiest way of checking if the amount of delay for each path is correct, is to take the impulse response of the system substituting the equalization stage by a simple delayed buffer corresponding to the same amount of delay that would have been introduced by that stage. This could be done using the simulator of the ADSP21020, for instance.

By taking the impulse response of the system with the equalization stage bypassed, an impulse should be reconstructed at the output. If this is not the case, one will perceive the impulse responses of some (if not all) the filters of both the QMF steps and the interpolation steps.

Misalignments and magnitudes of the responses present at the output could provide the clue for where in the system the delay given is wrong. If not, a re-inspection of the design by hand could easily be done, since all the filters provide a predictable delay.

With the equalization stage placed in the lowest frequency path, the impulse response of the entire system will be the impulse response of that stage, which should be different for each channel.

Some Issues on Performance

The system designed is intended to operate at a 16KHz sampling rate. Considering that the DSP runs at 33MHz, this allows for 2062 cycles between samples. We make use of 17-tap long filters for the QMF and reconstruction stages, and intend to use the same number of taps for the QMF bank of the equalization stage. The FFT-based FIR uses - for instance - a 256 point FFT/IFFT.

Using a 256 point FFT, there is not much improvement in the filtering process up to a 60 tap filter. That means the FFT-based FIR will be roughly as computationally efficient as a direct FIR, in terms of cycles per sample. For larger responses, the FFT-based approach is better. For instance, a 100 tap FIR filter implemented with an FFT length of 1024 points will spend 27 cycles per sample, while a direct FIR would roughly spend 108 cycles. A balance is still needed, however, between filter length, FFT length and memory requirements.

We need to perform two FFT-based FIR plus the QMF/decimation and interpolation. For a 200 tap filter (maybe small for our application), the number of cycles per output sample for each FFT-based FIR would be around 100, estimated from the benchmarks in [Meyer91]. We would have therefore 200 cycles per output sample only for the FFT-based FIR. This number seems very low, and this is due to the fact that the benchmarks measured took in to account only the cycles after the block of data had already been stored for processing.

For a 256 point FFT, one needs to store those blocks of samples prior to filtering. Since the FFT-based FIR filter works with a reduced sampling rate, one of every eight samples is actually input to the FFT-based filter, 2048 samples are taken (at the first QMF stage) before a block is sent to computation. This filtering scheme, however, is done much faster with those filters in comparison to direct time-domain FIR, and we have more time to perform the computation required at a reduced rate, so this process turns out to be advantageous for long filters.

From the tests performed with the equalization stage bypassed, it was observed that for three QMF banks and low pass interpolation filters of order 16, the reconstruction of an impulse input to the system happened 113 cycles later, indicating that for any sampling rate used in digital audio that group of operations can be done in between samples. As for computation load at those stages, the worst case is for every eighth sample, or the ones that is actually placed in the input buffer of the FFT-based filter.

These samples pass through four QMF banks, that for each input sample require 32 cycles to perform one low pass and one high pass filter on two channels. So for each sample input to the FFT-based filter, 128 cycles of filtering have already been spent. The code for these optimized filters is shown in Appendix C.

TABLE 5. Computation Load per Input Sample

Operation	Cycles (each channel)
QMF bank (each)	32 (128 worst case)
FFT-based filter (100 taps), 1024 points FFT (optimal)	24877.5/(1024-taps+1) = 26.9 [source Analog Devices]
Reconstruction Filters	30 (120 worst case)

There are more efficient ways of implementing the analysis/synthesis stages. The approach described made use of subroutine calling for every incoming sample for the QMF banks. Perhaps a more efficient way would be to use an input buffer to select the samples, and call only the necessary QMF bank according to the selection.

The final concern is memory requirements. The process involves four delayed buffers for the high frequency components of each stage, and depending on the size of the FFT-based filter, the sizes of these buffers will increase significantly.

Conclusion

In this chapter, a proposed implementation of the equalizer was presented. Tests were done on a DSP-based platform. The programming of the parts involved was done in assembly language for the Analog Devices ADSP21020 floating-point signal processor.

Although a complete experiment was not possible to be conducted (to date) the major parts of the program implementing the equalizer are realizable. However, the coefficients of the equalizer need to be calculated and input to the FFT-based FIR filter as transfer functions for each channel. The system proposed is expandable and capable of performing other tasks such as filtering of the other frequency bands, if needed. The work demonstrates that our algorithm can be implemented in real-time on a commercial DSP.

CHAPTER 6 Conclusion and Future Work

Conclusion

A novel approach for multipoint room equalization has been presented. Results from simulation and an implementation attempt indicated the feasibility of a real-time system to perform the equalization of frequencies up to 500Hz in small venues, at multiple listening positions.

Results from simulation clearly indicate the feasibility of the multipoint room equalization algorithm and although hardware improvement is still to be done, the algorithm has been proven to be feasible and realizable. A real-time implementation has been proposed using a DSP-based platform to evaluate how efficient the simulated algorithm is. This implementation made use of the main configuration described in this thesis, which is a 2 speaker, 2 listening positions system.

This thesis has made contributions in the following aspects:

1. The approach for the solution of the inverse filtering problem applied to room equalization at multiple listening positions has shifted the solution from inverting each transfer function to inverting a matrix of transfer functions. This novel algorithm has been proposed and tested. The algorithm identifies and fixes features at frequencies that indicate difficulties for calculating the inverse of a matrix of transfer functions;

2. A new version of the inverse of a matrix of transfer functions working in conjunction with a set of desired gains has been presented and explained, and the results compared with the ones obtained using the natural inverse of the transfer function matrix;

3. A real-time implementation of the equalizer has been proposed. For the real-time implementation of the equalizer, multirate signal processing techniques were used as well as frequency domain FIR filtering. These techniques were proven to allow for a very efficient implementation, making use of fast, small FIR filters to perform the band split, working in conjunction with a fast FFT-based FIR filter performing the equalization.

Future Work

Many details to improve or expand the algorithm presented were pointed out through the thesis as being left for future work. Here we list the most important steps for future work.

The first step is to conduct an experiment in a small reverberant venue, and upgrade the real-time implementation (hardware and code), as to allow for longer room transfer functions. The background work is presented in this thesis, and this would be just an expansion of the work here presented.

An investigation of the 3x3 case, particularly on the adjustment of the phase for the desired set of complex gain is necessary. A good understanding of that case will lead to expanding the system to a 4x4 case, which most likely will fall into the same category as far as choosing an appropriate delay. A 4x4 case would be ideally applied to equalizing the overall transfer function at the four "main" positions in a car.

An interesting follow-up to this thesis is an investigation of non-square matrices of transfer functions, with the number of drivers greater than the number of listening positions. This would require taking the inverse of a non-squared matrix and point towards Singular Value Decomposition.

A new DSP based target platform could be designed for bigger systems. For instance, a system with three sound sources equalizing three listening positions. As to improving the algorithm, research is still to be done in managing stereo signals. This would directly imply an application for multipoint equalization in home theatres. No research has been done in this thesis with regard to that problem. The secret seems to lie in a matrix of desired gains as opposed to a vector of desired gains, the case investigated.

Finally, since the design of new signal processors is pointing towards multiprocessing, a multiprocessor version of the algorithm here presented could be implemented. Such an implementation would be able to handle different types of signals and a variable number of positions to be equalized.

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JASA - Journal of the Acoustical Society of America

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APPENDIX A

Measurements for Different System Configurations

Introduction

This Appendix presents the acoustical data measured in a car. There are 16 impulse responses that are grouped in sets of 2x2, 3x3 and 4x4. For each group, all combinations of loudspeakers and listening positions are explored. Sets of impulse responses, frequency responses are presented for the 2x2 system only, since this configuration is the one explored more thorougly in this thesis. For 3x3 and 4x4 systems, only the condition number versus frequency plots are presented. The plots related to the condition number for 2x2 systems are presented in Appendix B to complete the results of the equalization process.

System Configurations

The following plots were generated by combining impulse response measurements inside a car, representing the paths between listening positions (microphones) and sound sources (loudspeakers) in groups of two.

Configurations for 2 positions

The sets are shown as individual groups of four plots, resembling a 2x2 matrix, with the colums representing the loudspeakers related to the positions involved and the rows representing the listening positions (microphones). The sets are shown in this way to resemble the configuration of the transfer function matrix H for each case. The next figures show impulse responses and frequency response for 2x2 systems.

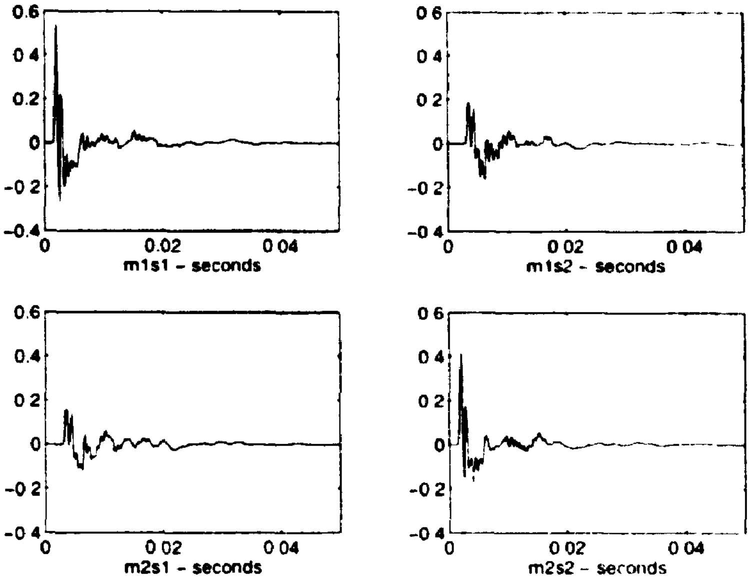

FIGURE 32. Impulse responses for 2x2 system - positions 1 and 2 using sources 1 and 2

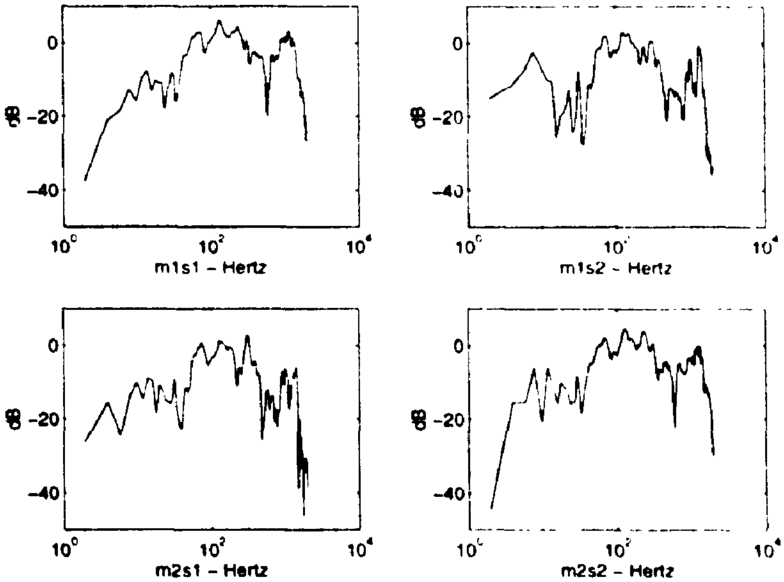

FIGURE 33. Frequency responses for 2x2 system - positions 1 and 2 using sources 1 and 2

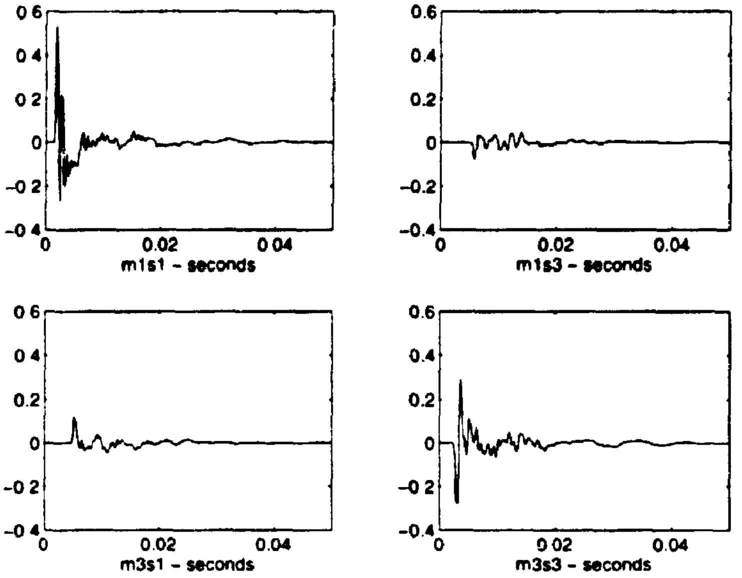

FIGURE 34. Impulse responses for 2x2 system - positions 1 and 3 using sources 1 and 3

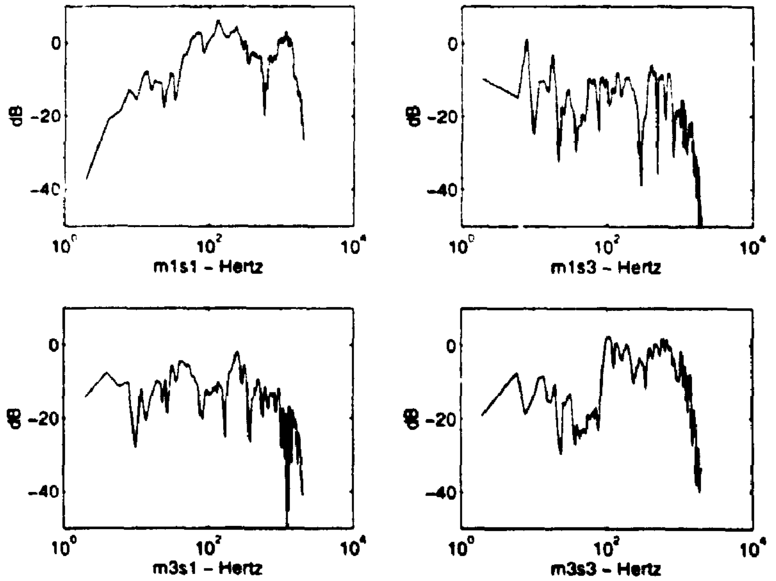

FIGURE 35. Frequency responses for 2x2 system - positions 1 and 3 using sources 1 and 3

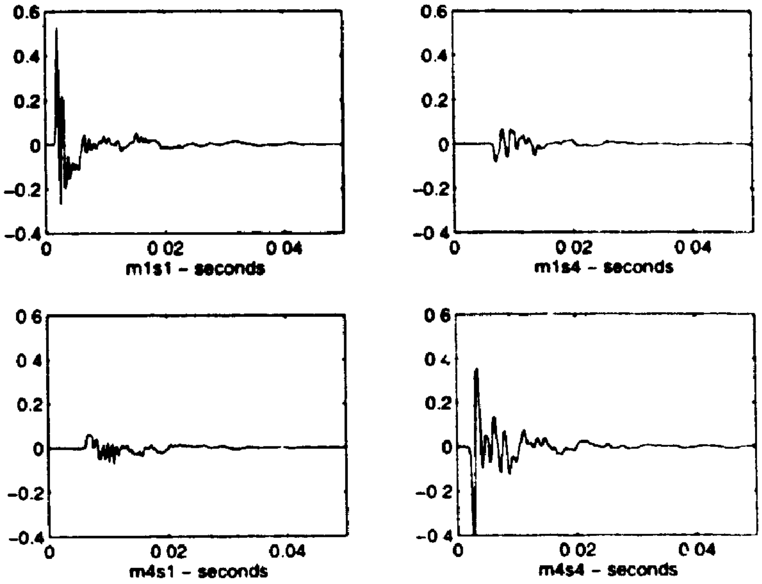

FIGURE 36. Impulse responses for 2x2 system - positions 1 and 4 using sources 1 and 4

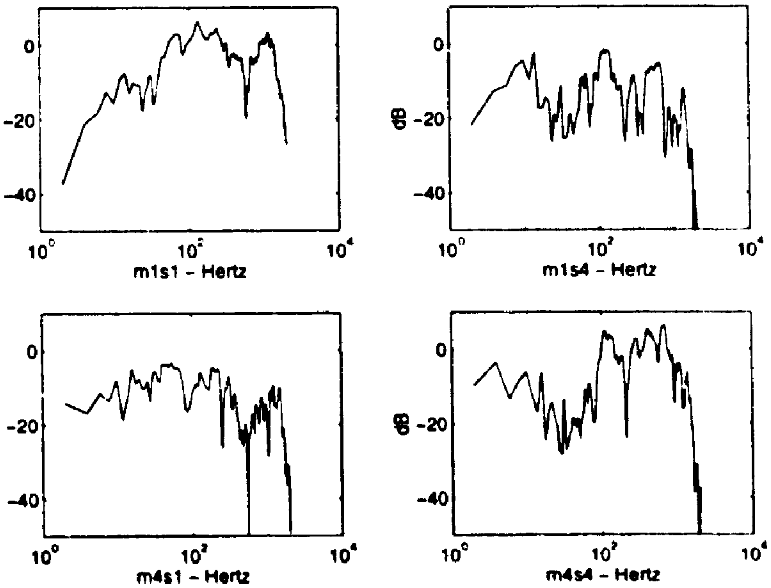

FIGURE 37. Frequency responses for 2x2 system - positions 1 and 4 using sources 1 and 4

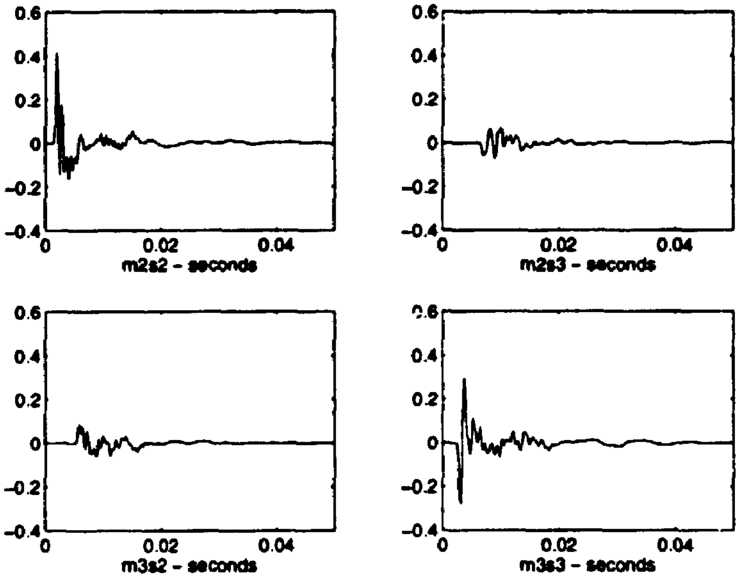

FIGURE 38. Impulse responses for 2x2 system - positions 2 and 3 usiing sources 2 and 3

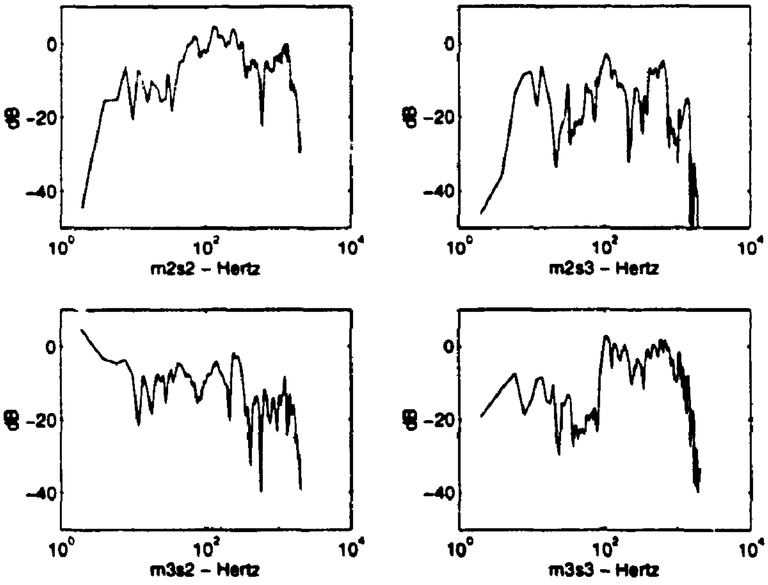

FIGURE 39. Frequency responses for 2x2 system - positions 2 and 3 using sources 2 and 3

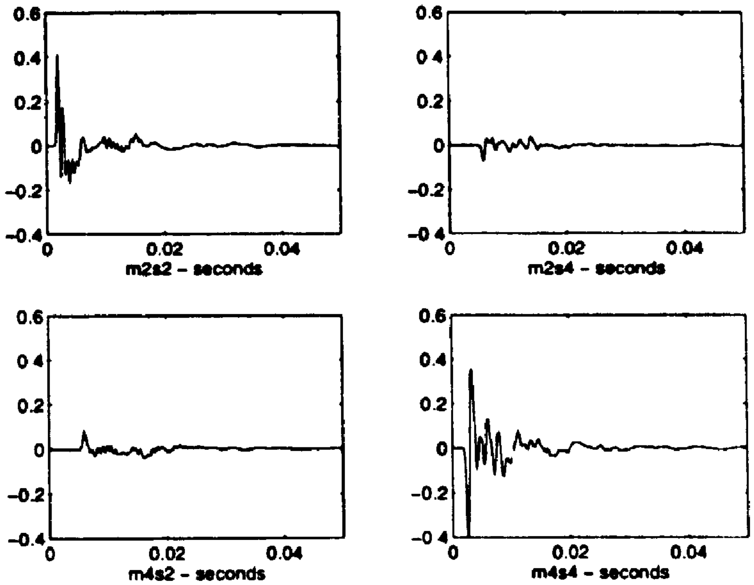

FIGURE 40. Impulse responses for 2x2 system - positions 2 and 4 using sources 2 and 4

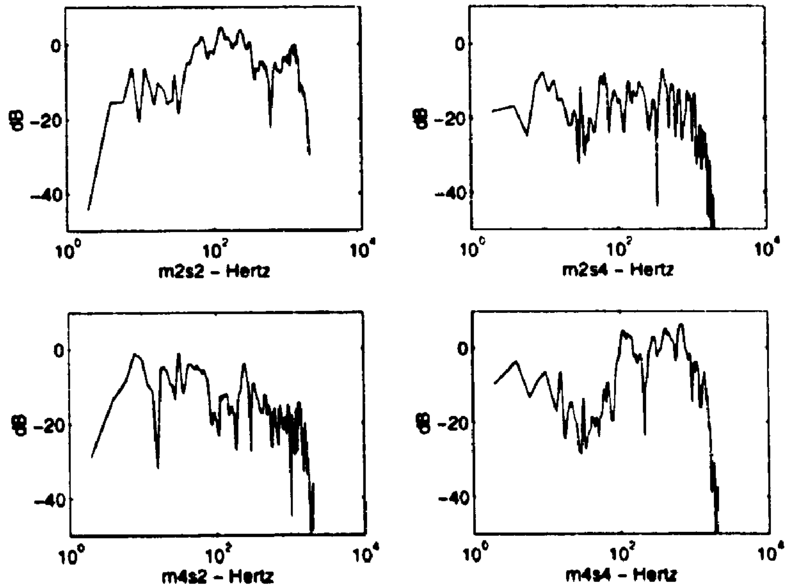

FIGURE 41. Frequency responses for 2x2 system - positions 2 and 4 using sources 2 and 4

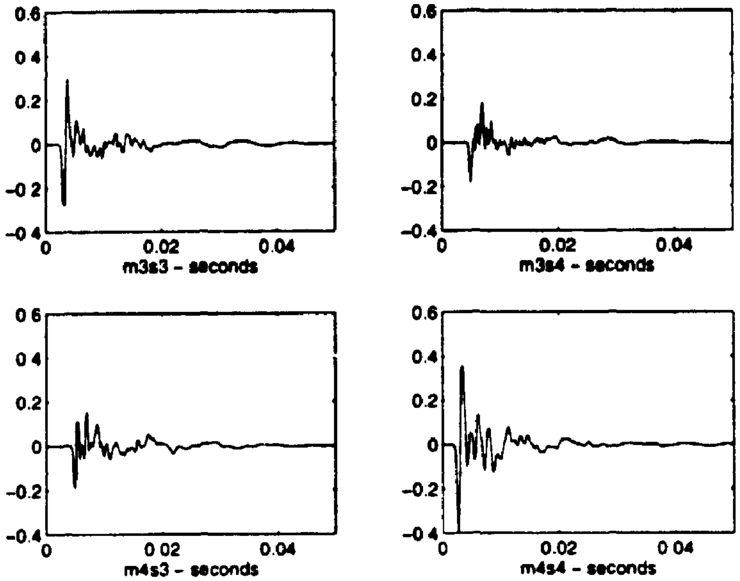

FIGURE 42. Impulse responses for 2x2 system - positions 3 and 4 using sources 3 and 4

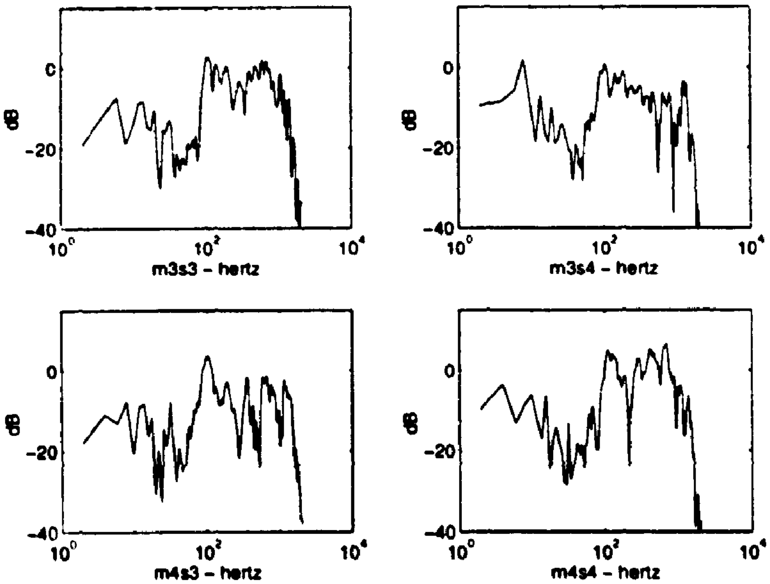

FIGURE 43. Frequency responses for 2x2 system - positions 3 and 4 using sources 3 and 4

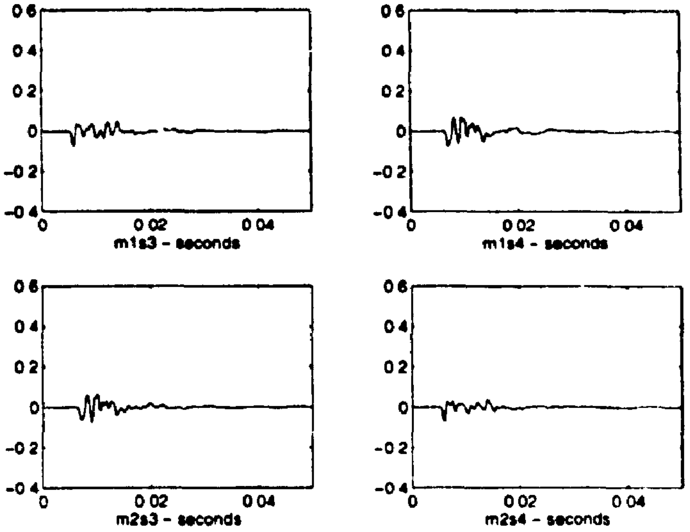

FIGURE 44. Impulse responses for 2x2 system - positions 1 and 2 using sources 3 and 4

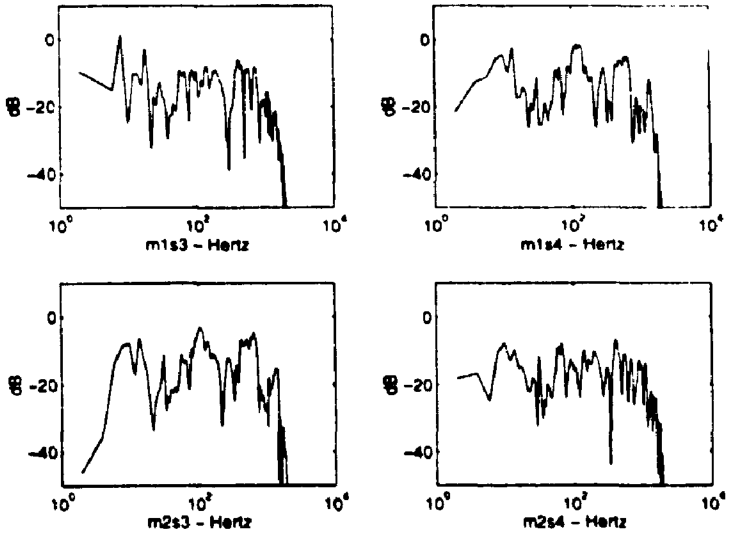

FIGURE 45. Frequency responses for 2x2 system - positions 1 and 2 using sources 3 and 4

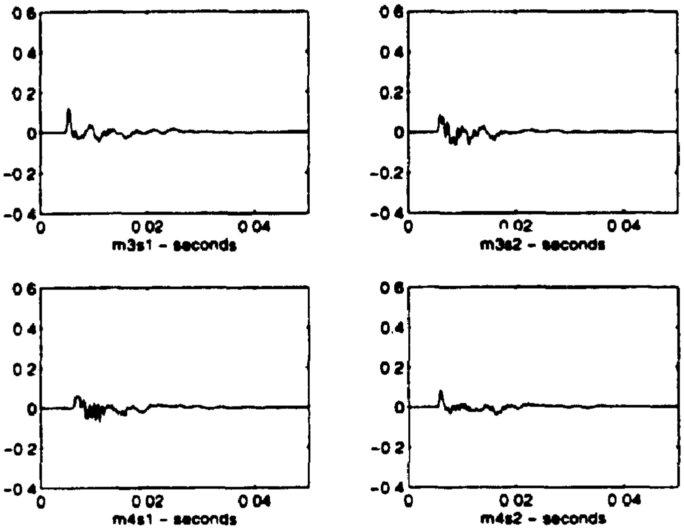

FIGURE 46. Impulse responses for 2x2 system - positions 3 and 4 using speakers 1 and 2

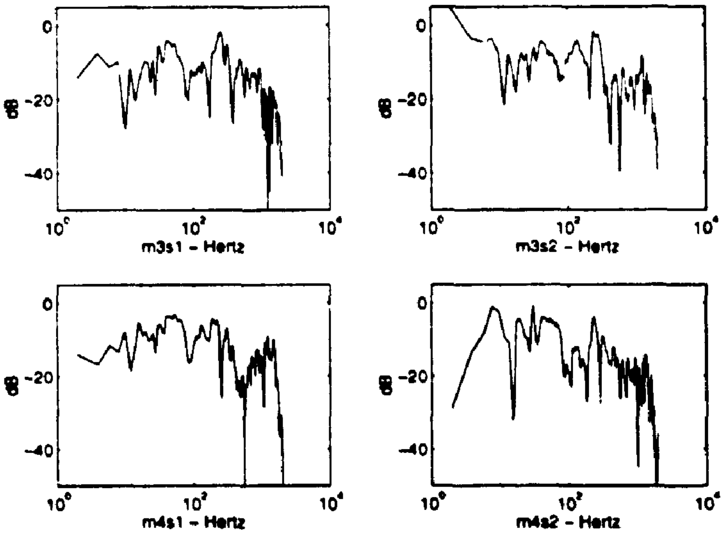

FIGURE 47. Frequency responses for 2x2 system - positions 3 and 4 using sources 1 and 2

Configuration for 3 and 4 positions - Condition Number plots

In the following graphs, only the condition number will be presented versus frequency, for the impulse and frequency responses have already been shown previously.

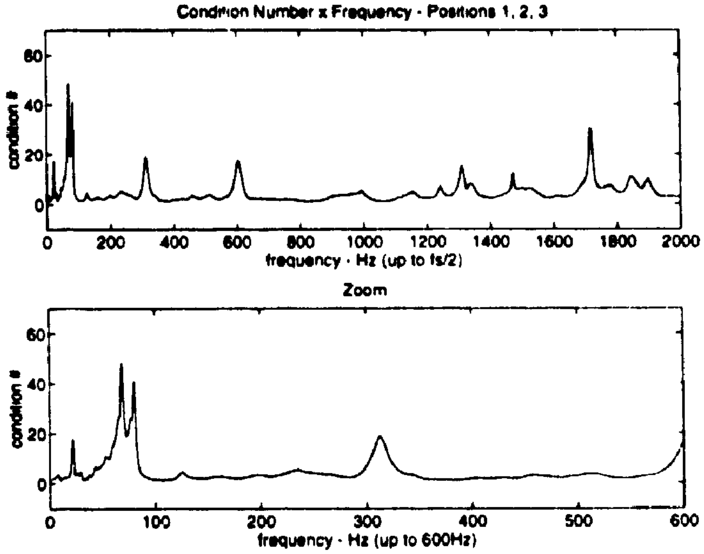

FIGURE 48. Condition Number versus frequency for 3x3 configuration - positions 1, 2 and 3

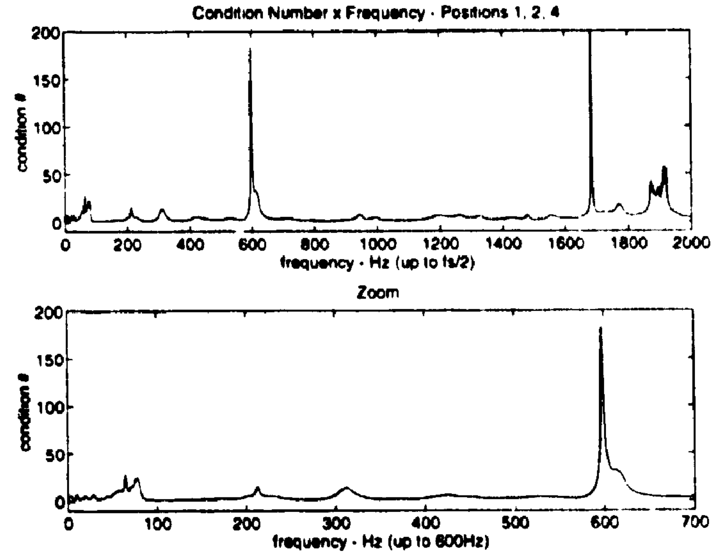

FIGURE 49. Condition Number versus frequency for 3x3 configuration - positions 1, 3 and 4

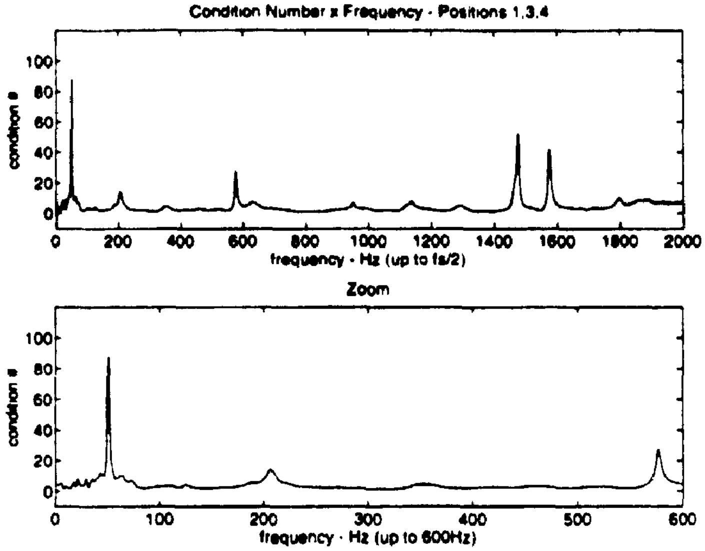

FIGURE 50. Condition Number versus frequency for 3x3 configuration - positions 1, 3 and 4

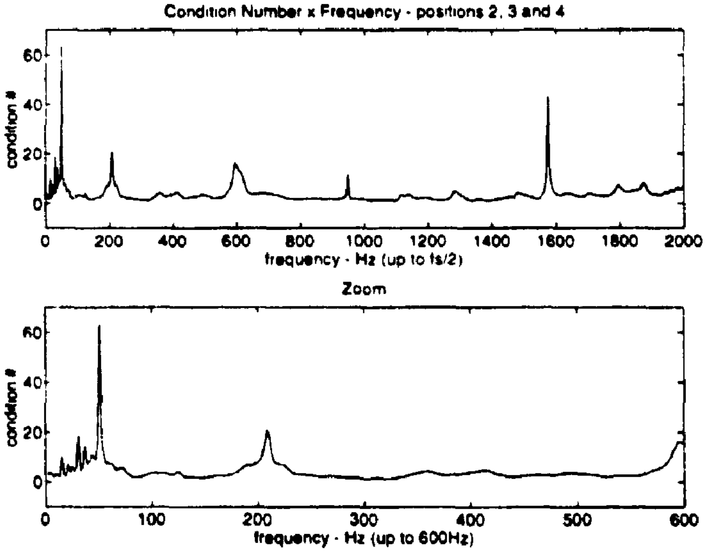

FIGURE 51. Condition Number versus frequency for 3x3 configuration - positions 2, 3 and 4

Finally, the condition number versus frequency plot is shown for a system with four sound sources used for equalizing the response at four listening positions.

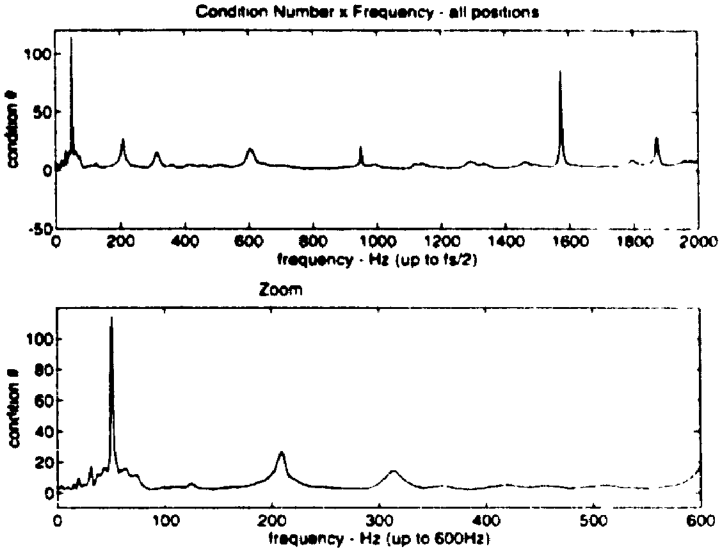

FIGURE 52. Condition Number versus frequency for 4x4 configuration - positions 1, 2, 3 and 4

APPENDIX B

Equalization Results

Introduction

This appendix contains results obtained after applying the equalization algorithm to different combinations of loudspeakers and listening positions. The combinations were concocted from measurements collected in a car.

The equalization procedure was applied to all combinations of two loudspeakers driving two listening positions for the set of 16 transfer functions measured. The most important features from the results of each set are highlighted. The results for the two combinations presented in chapter 4 are excluded from this appendix.

Combinations of Sources and Listening Positions

This section provides plots comparing the direct inverse of the transfer function matrix and the new version of the inverse, calculated by the equalization algorithm. The plots are presented for each element of the natural inverse and new inverse matrices. This is done in order to show the improvement achieved, particularly at the frequencies where a peak in the condition number has been identified.

Following these results for each of the combination of listening position-sound source, the results of the equalization procedure are shown. It is noticed in those results the flatness achieved for the overall response, and the linearity of the phase for the band of interest, which is from 50 to 500Hz.

The plot of the condition number versus frequency was also provided in order for the reader to identify the peaks in the condition number and visualize the artifacts present in the elements of the direct inverse of the transfer function matrix for the frequencies presenting the peaks.

Listening Positions 1 and 3 Using Sound Sources 1 and 3

This combination of sound sources/listening positions presents a bigger peak in the condition number at around 70Hz. The frequency was the one used as the "constraining" frequency. The whole range of frequencies from 60Hz to 80Hz presents high condition numbers and two peaks are more noticeable. Only the highest peak was used in the algorithm, since there is a limit of one frequency being used in the correction of the others.

It is interesting to point out that the results show the presence of peaks in the elements of the new inverse of the transfer function matrix at the frequency where the second peak is located (around 80Hz). This peak in the elements of the new inverse would have been brought down if it were possible to use the constraints imposed by the frequency where the second peak in the condition number is located.

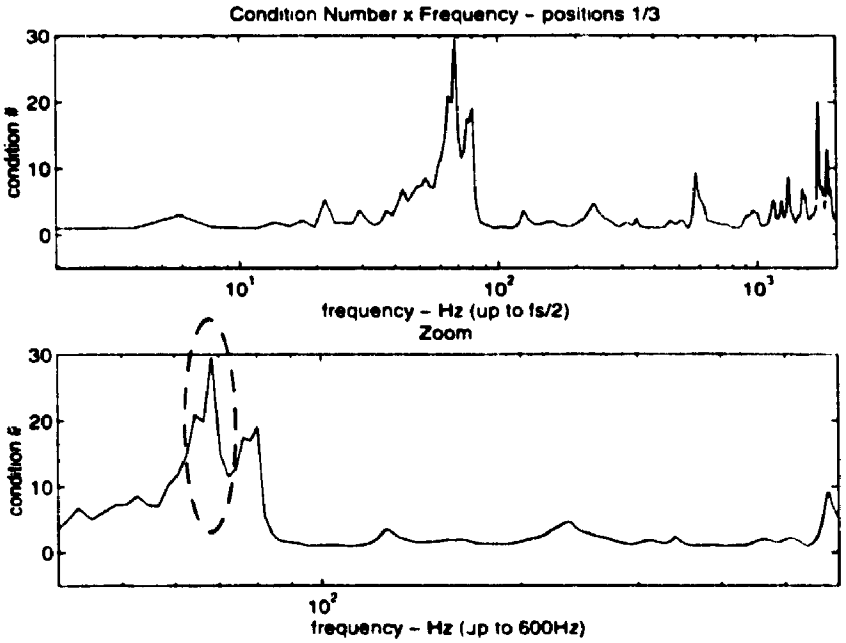

FIGURE 53. Condition Number versus frequency - positions 1 and 3

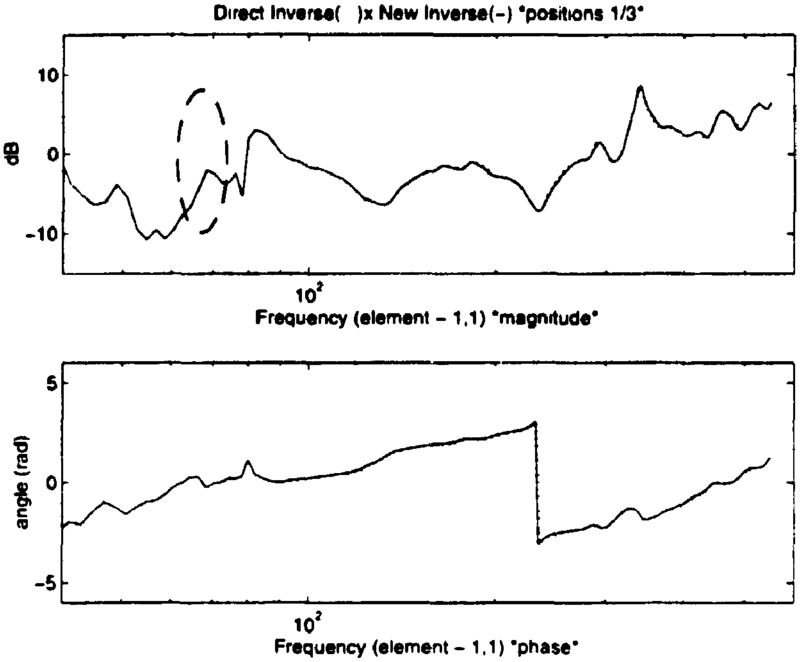

FIGURE 54. Direct inverse of H and New inverse of H - element 1,1 (equalization of positions 1 and 3)

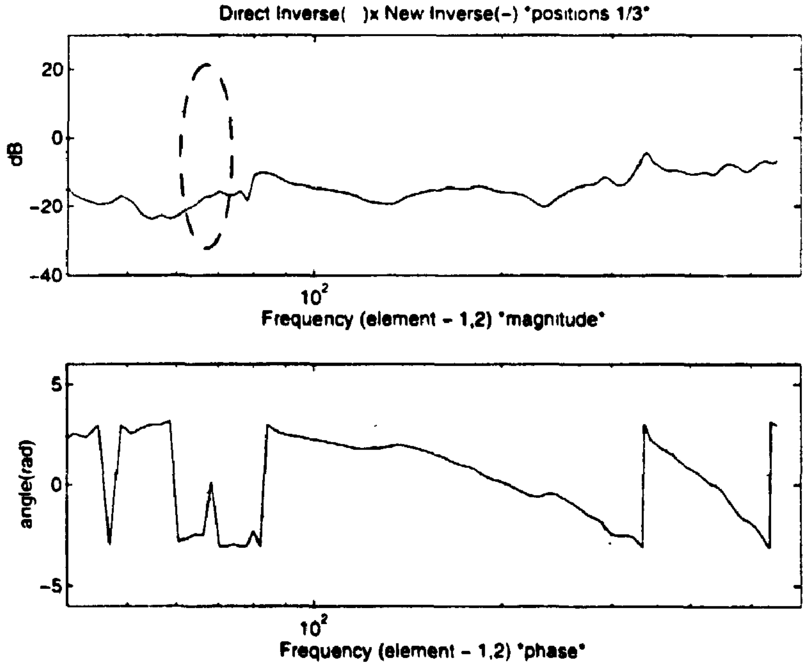

FIGURE 55. Direct inverse of H and New inverse of H - element 1,2 (equalization of positions 1 and 3)

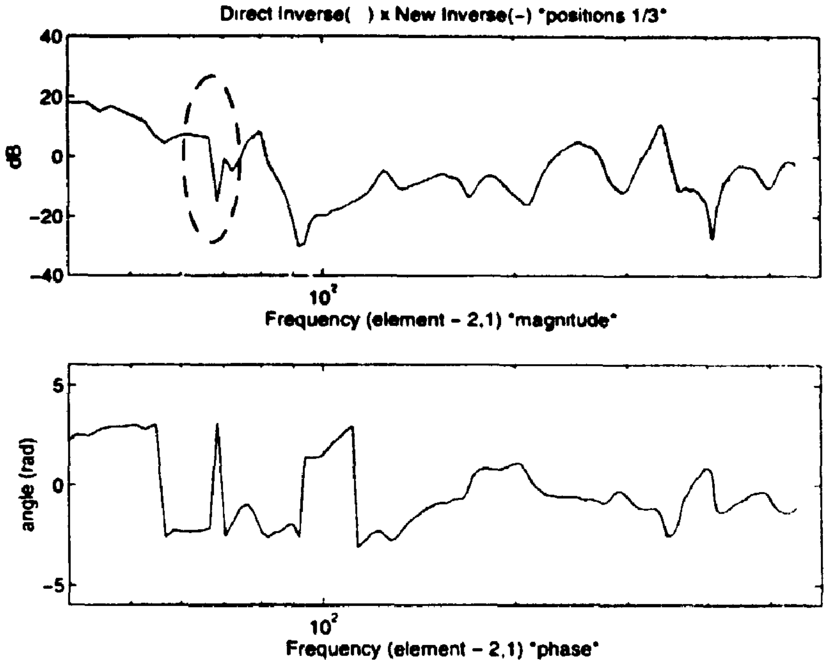

FIGURE 56. Direct inverse of H and New inverse of H - element 2,1 (equalization of positions 1 and 3)

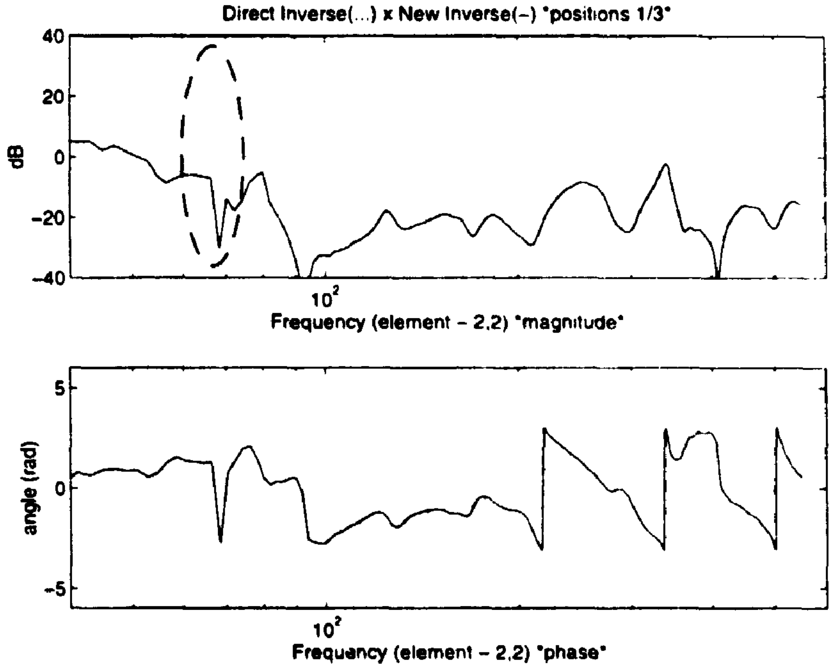

FIGURE 57. Direct inverse of H and New inverse of H - element 2,2 (equalization of positions 1 and 3)

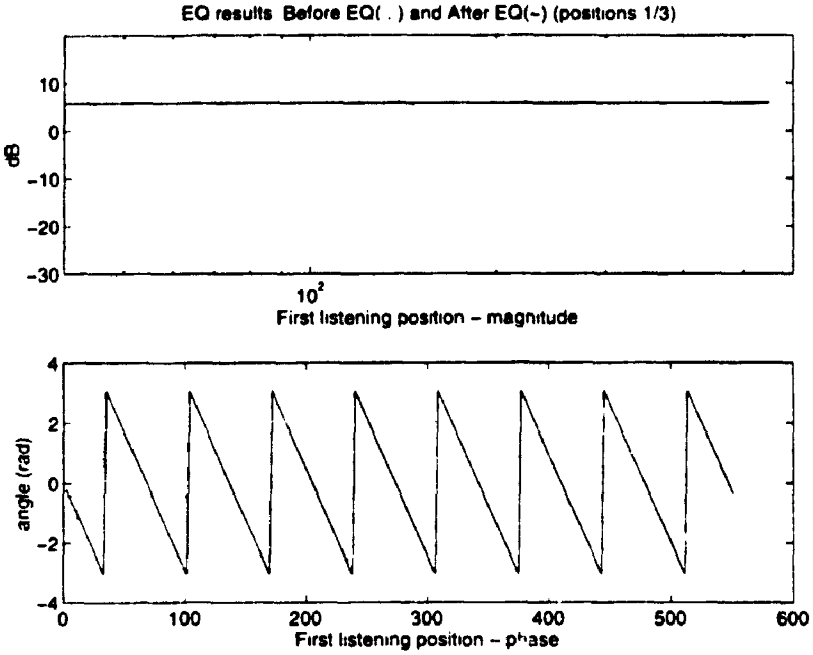

FIGURE 58. Equalization result for position 1 - using sources 1 and 3

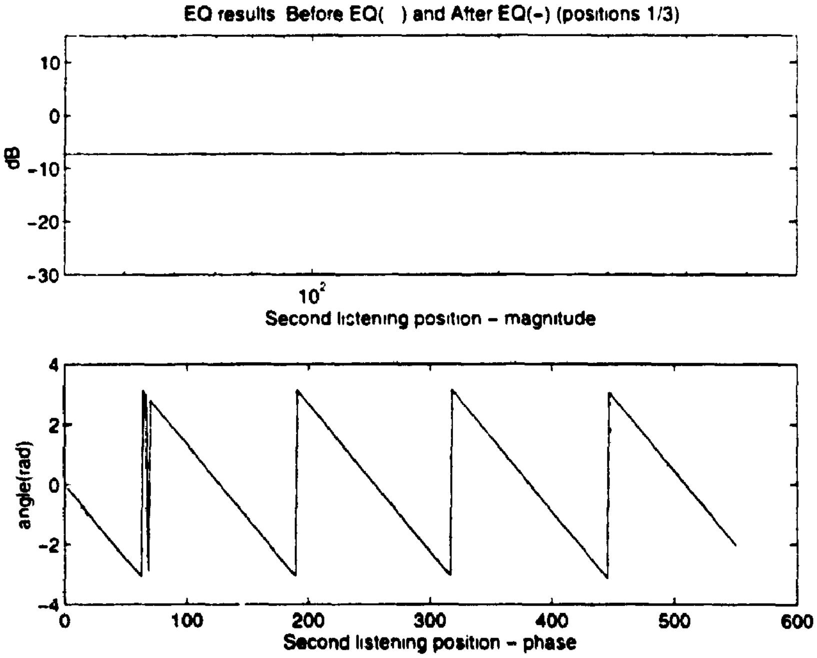

FIGURE 59. Equalization result for position 3 - using sources 1 and 3

Listening Positions 1 and 4 Using Sound Sources 1 and 4

For this case, the frequency used was the one corresponding to the peak in the condition number at around 200Hz. Once again there were other peaks with high condition numbers within the band of interest. The frequency with the biggest condition number was chosen.

The graphs of the direct inverse and the new inverse of the transfer function matrix show the features associated with the peaks present at approximately 80Hz and 60Hz. There are peaks for each element of the inverses caused by these bad condition numbers.

For the frequency chosen for the correction (200Hz), improvements are noticeable between the direct inverse of the transfer function matrix H and its new inverse calculated by the algorithm.

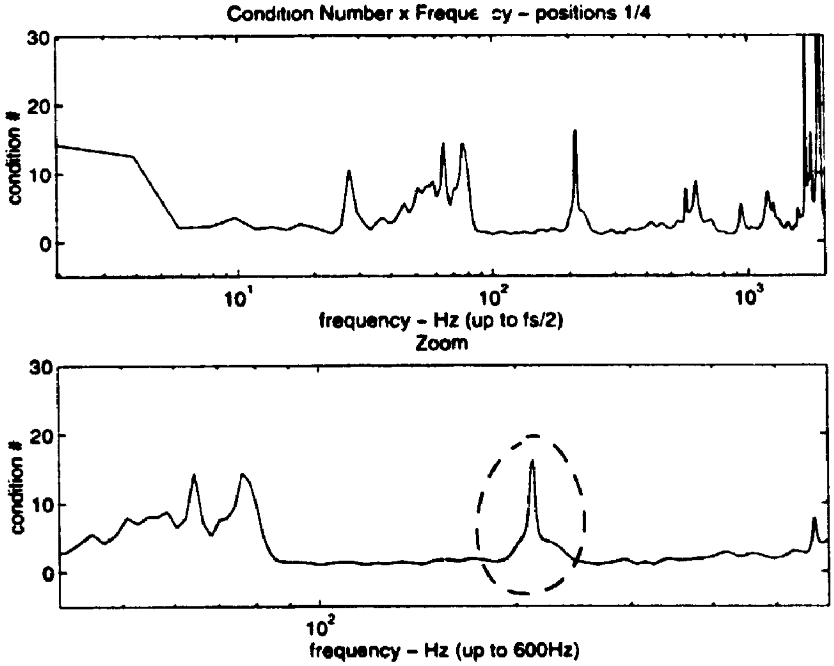

FIGURE 60. Condition Number versus frequency - positions 1 and 4

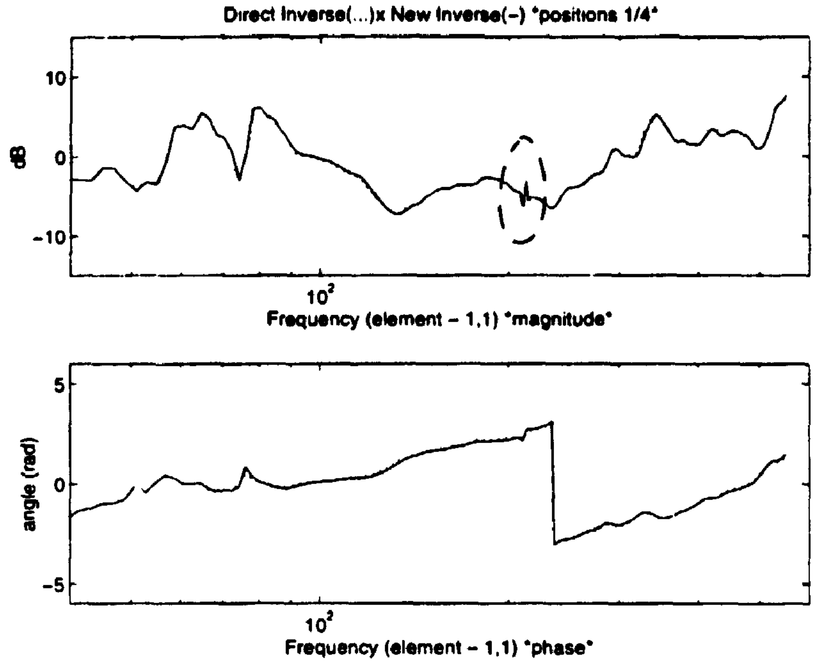

FIGURE 61. Direct inverse of H and New inverse of H - element 1,1 (equalization of positions 1 and 4)

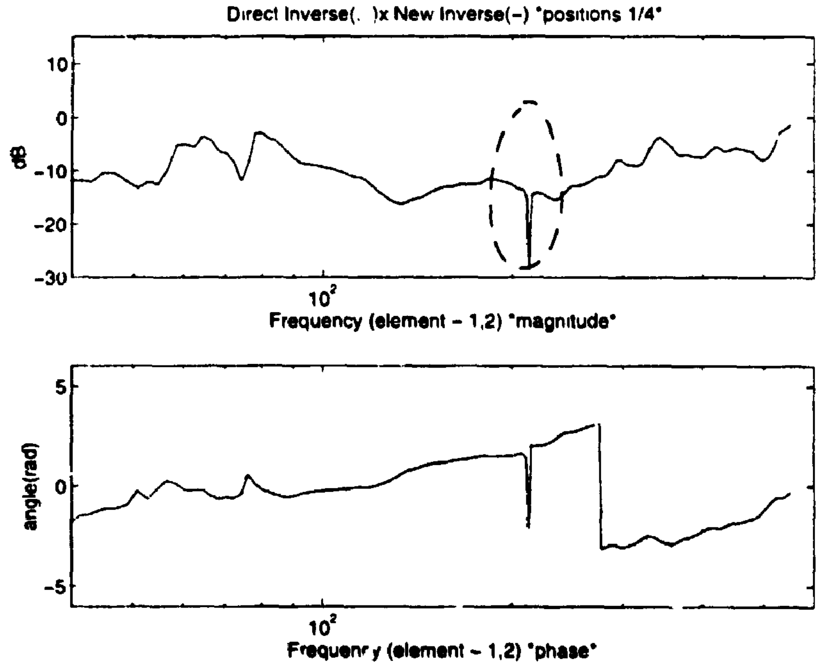

FIGURE 62. Direct inverse of H and New inverse of H - element 1,2 (equalization of positions 1 and 4)

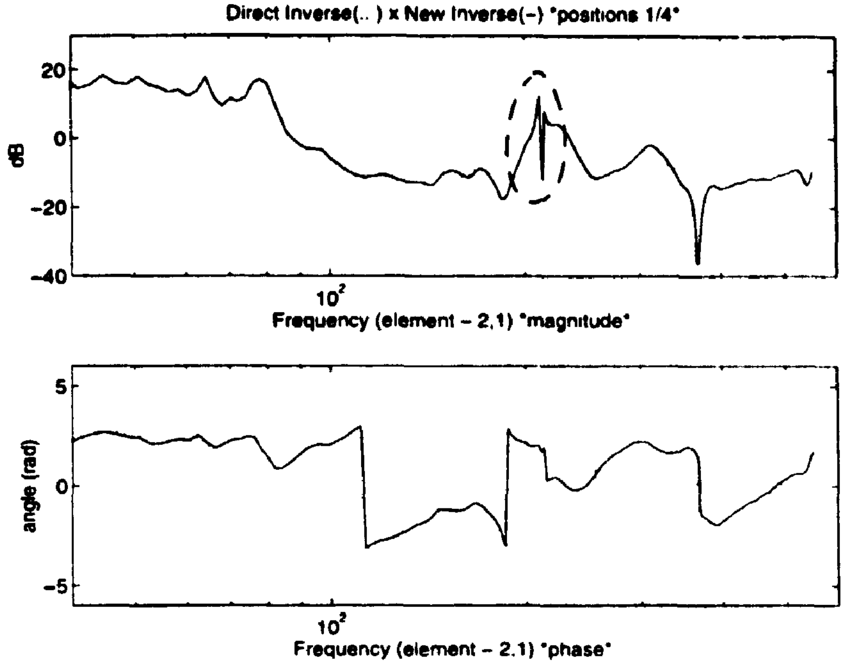

FIGURE 63. Direct inverse of H and New inverse of H - element 2,1 (equalization of positions 1 and 4)

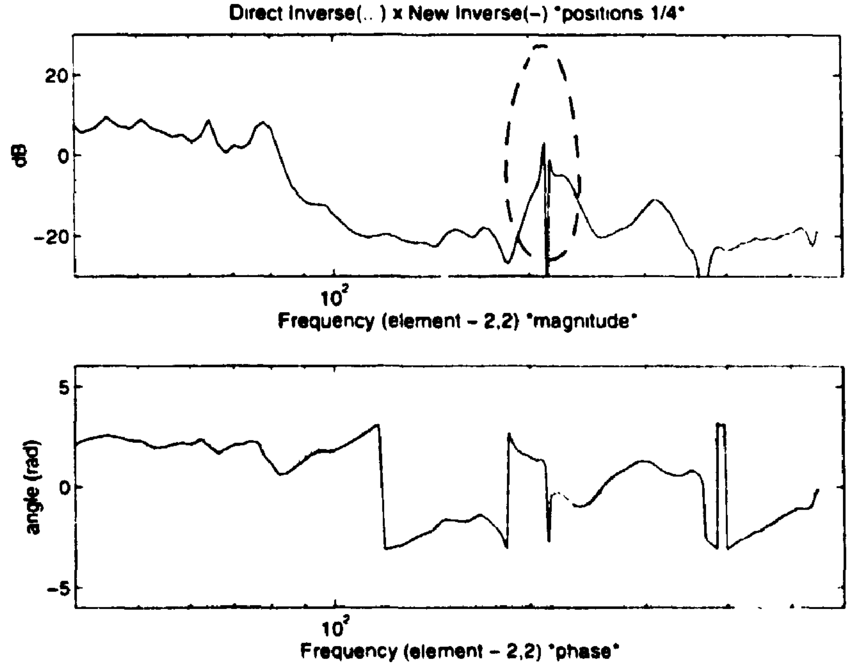

FIGURE 64. Direct inverse of H and New inverse of H - element 2,2 (equalization of positions 1 and 4)

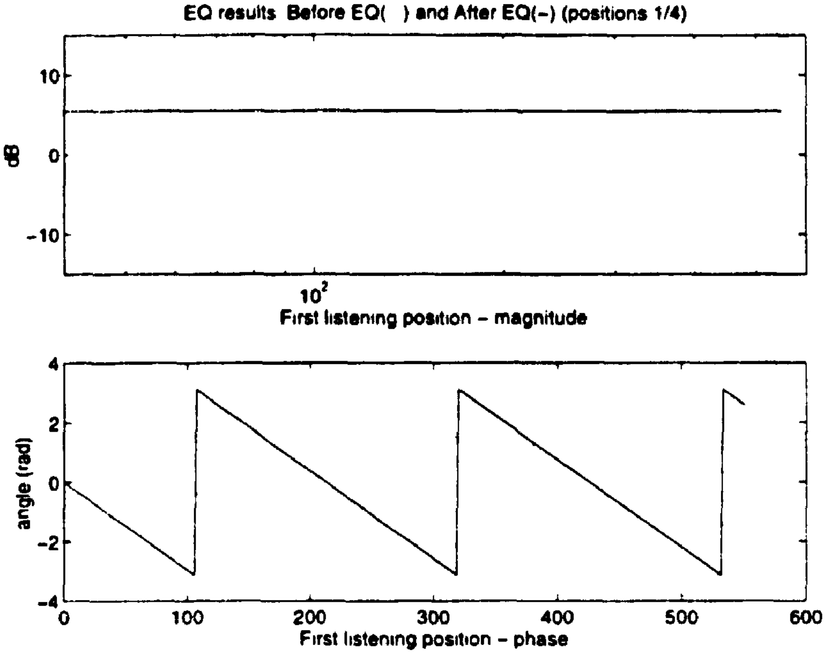

FIGURE 65. Equalization result for position 1 - using sources 1 and 4

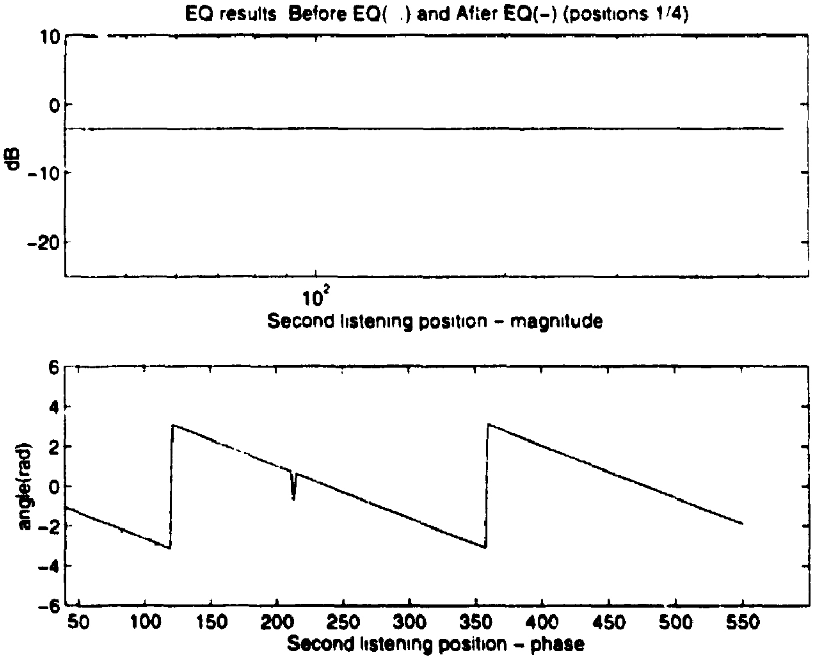

FIGURE 66. Equalization result for position 4 - using sources 1 and 4

Listening Positions 2 and 3 Using Sound Sources 2 and 3

In this case there is a peak clearly higher than all others, at around 80Hz. It is interesting to perceive the presence of peaks at that frequency for all configurations analyzed so far. This seem to consistently indicate the presence of a problematic frequency at 80Hz for that venue.

The results show great improvement for each element of the new inverse, particularly at the frequency where the peak in the condition number had been identified.

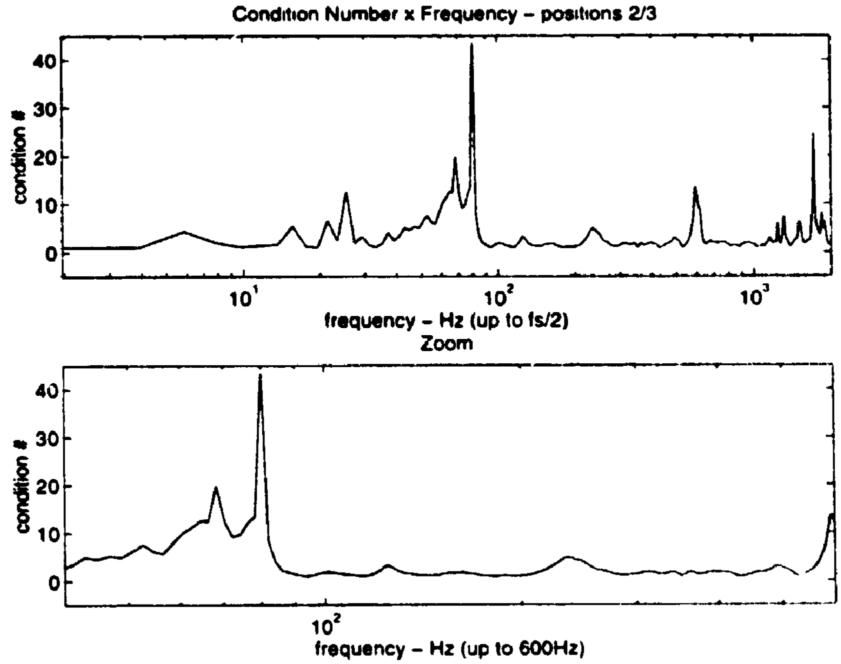

FIGURE 67. Condition Number versus frequency - positions 2 and 3

A small feature was observed in the results of the equalization, in both magnitude and phase, only at the frequency used as reference. This is believed to be due to the lack of "fine tuning" for the desired vector initially chosen. Another detail observed was the great difference in overall gain from one position to the other. This however, can be compensated after the equalizer.

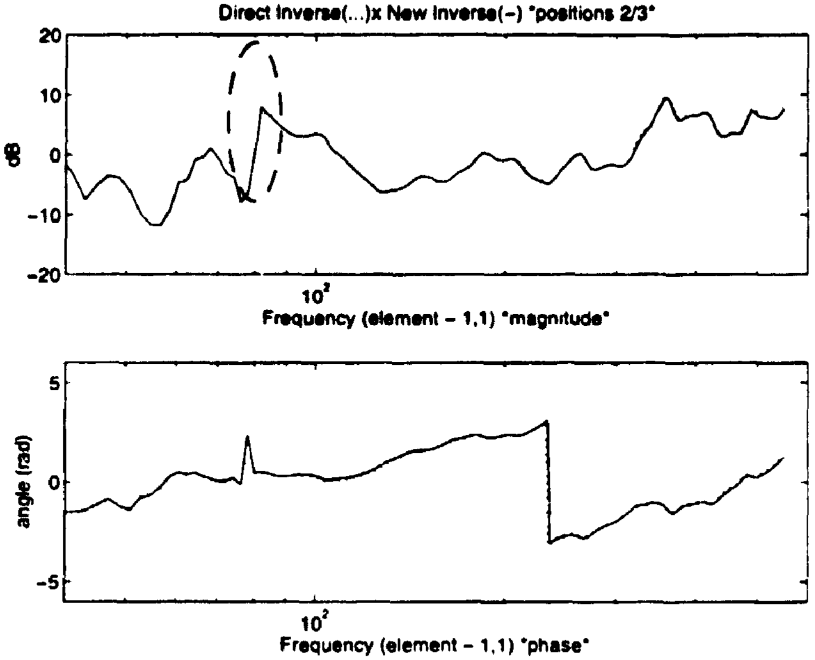

FIGURE 68. Direct inverse of H and New inverse of H - element 1,1 (equalization of positions 2 and 3)

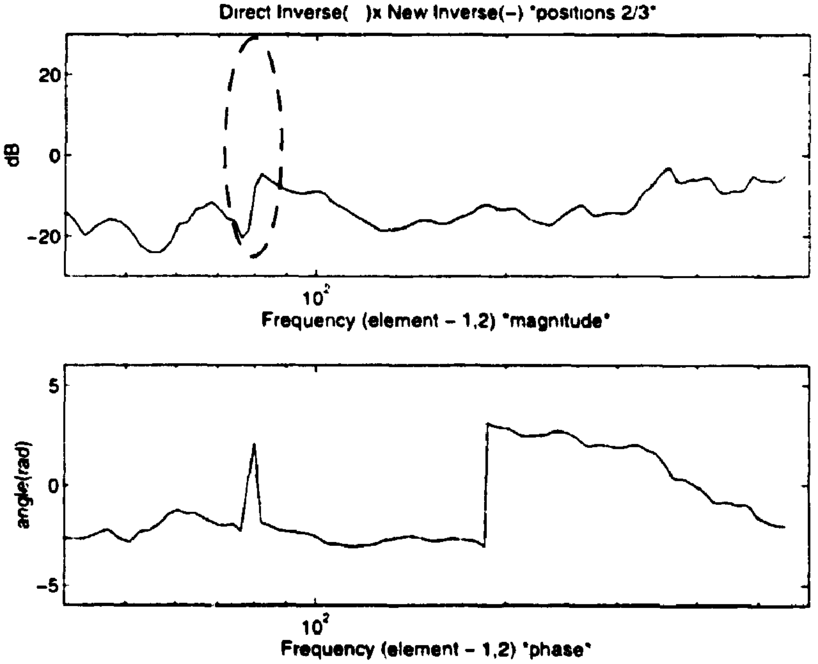

FIGURE 69. Direct inverse of H and New inverse of H - element 1,2 (equalization of positions 2 and 3)

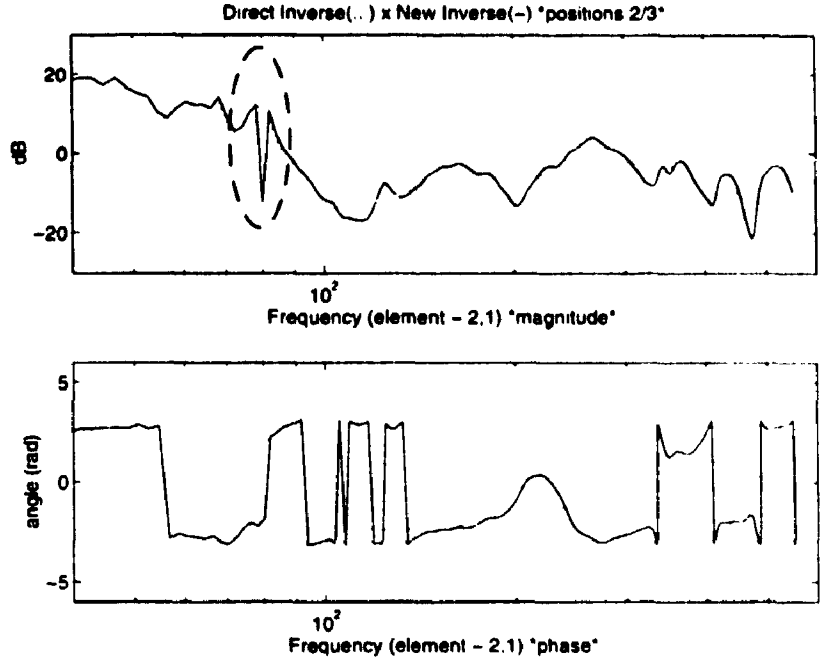

FIGURE 70. **Direct inverse of H and New inverse of H - element 2,1 (equalization of positions 2 and 3)**

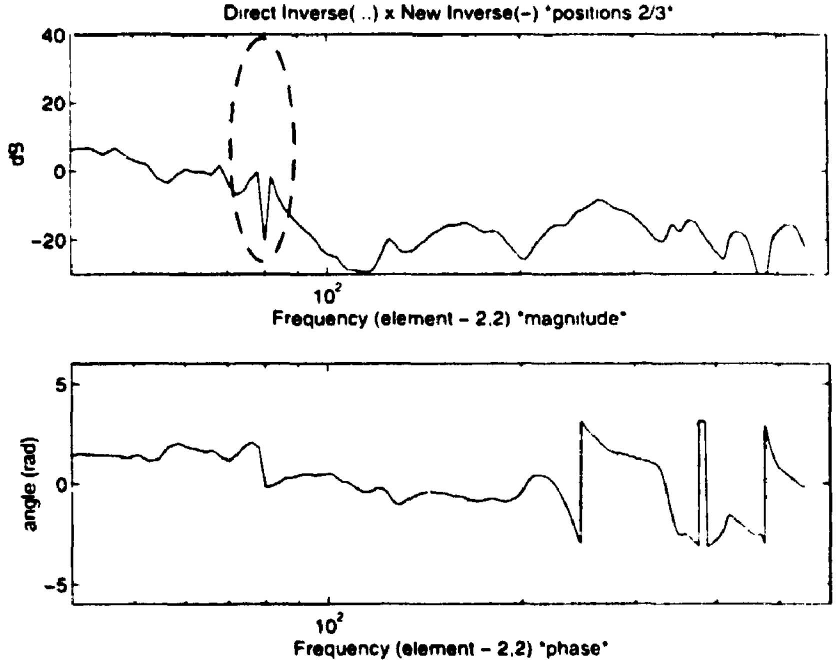

FIGURE 71. **Direct inverse of H and New inverse of H - element 2,2 (equalization of positions 2 and 3)**

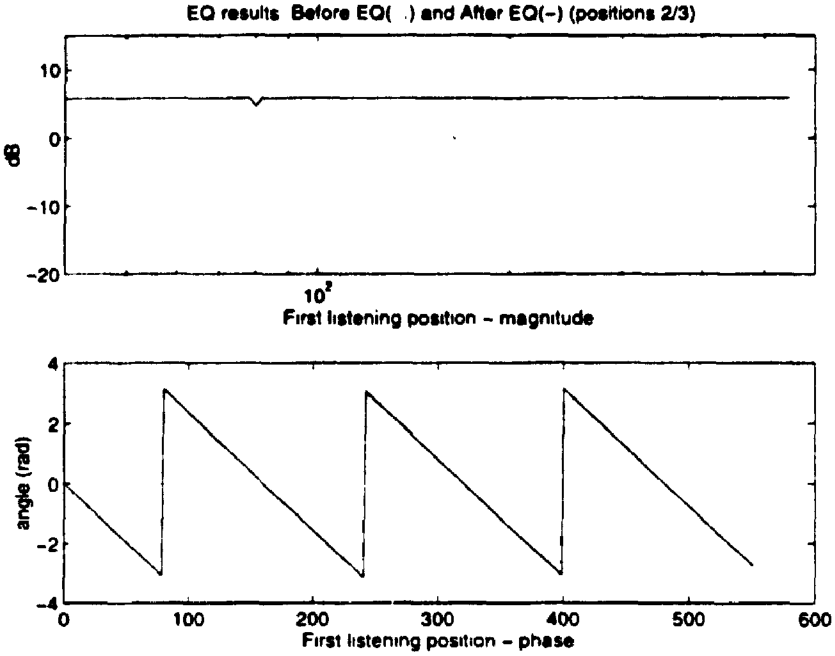

FIGURE 72. Equalization result for position 2 - using sources 2 and 3

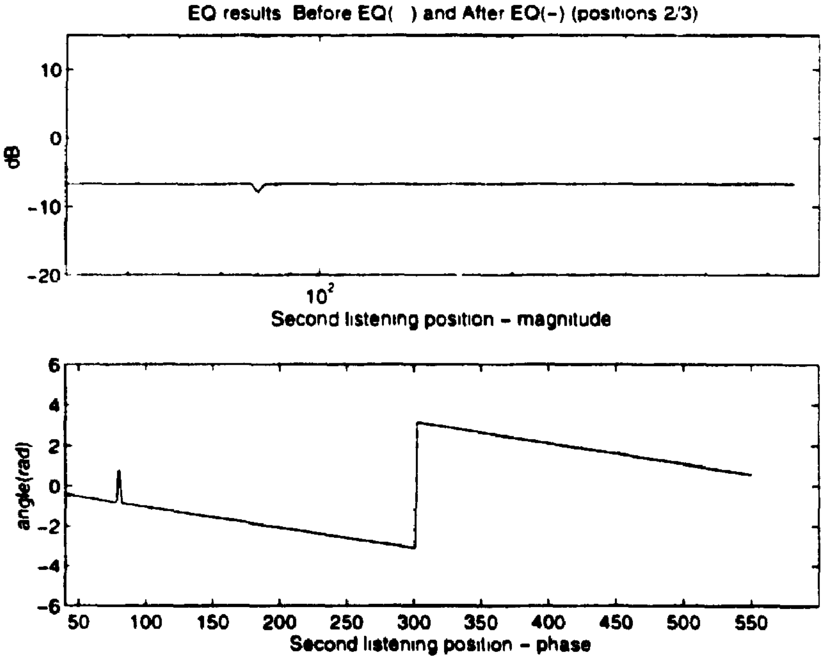

FIGURE 73. Equalization result for position 3 - using sources 2 and 3

Listening Positions 2 and 4 Using Sound Sources 2 and 4

In this particular case, the first peak in the condition number occurred for the very first "bin" of frequency. This peak was not used as a reference. Instead, the peak at around 65Hz was used in order to achieve the desired vector. This decision was made in order to have a frequency within the band of interest as a reference, rather than an inaudible frequency. Taking a look at the results of the equalization, one also notices - similarly to the previous case - the presence of a feature in the magnitude response at the frequency used as reference.

This was the configuration that presented most problems for the equalization. The small peak present even after the equalization could not be removed with the desired vector chosen. "Tuning" the desired vector is material for further research.

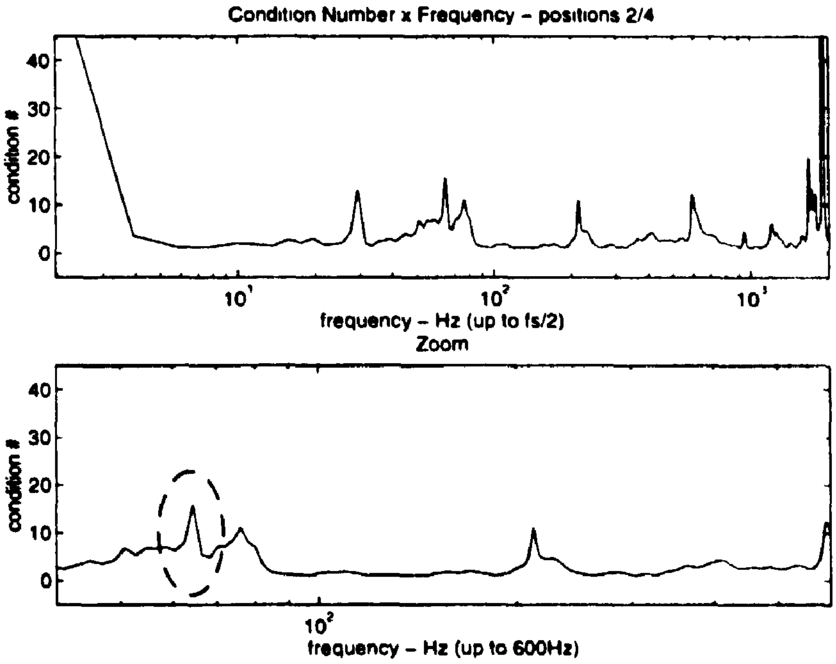

FIGURE 74. Condition Number versus frequency - positions 2 and 4

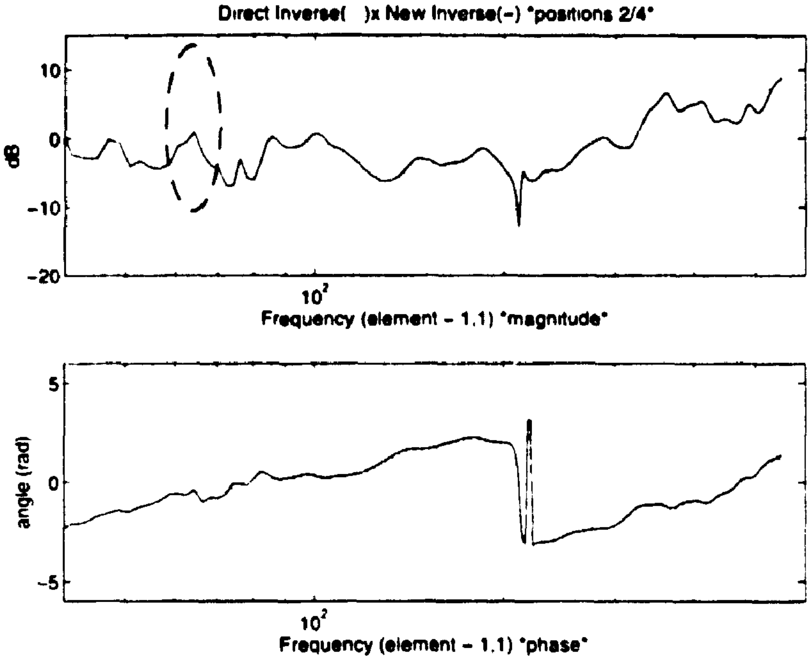

FIGURE 75. **Direct inverse of H and New inverse of H - element 1,1 (equalization of positions 2 and 4)**

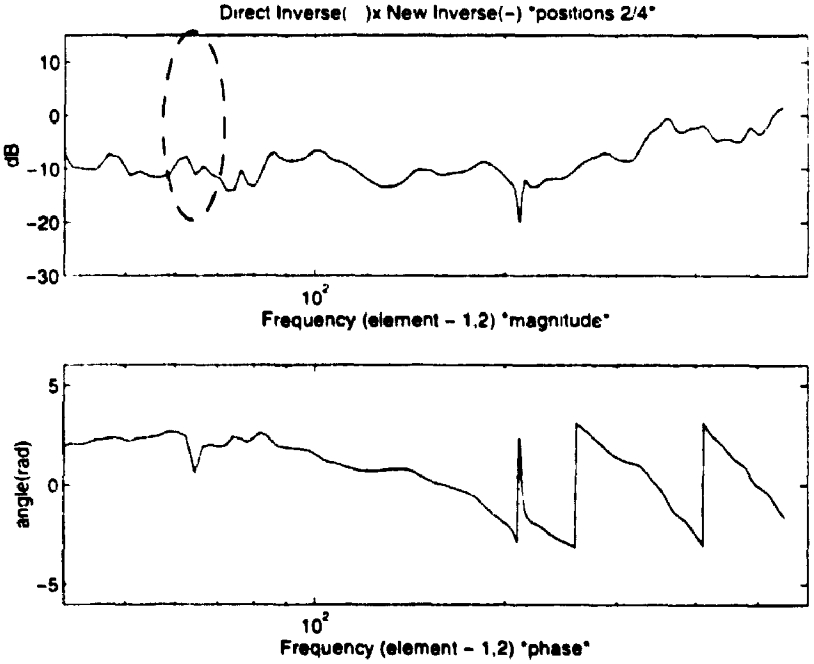

FIGURE 76. **Direct inverse of H and New inverse of H - element 1,2 (equalization of positions 2 and 4)**

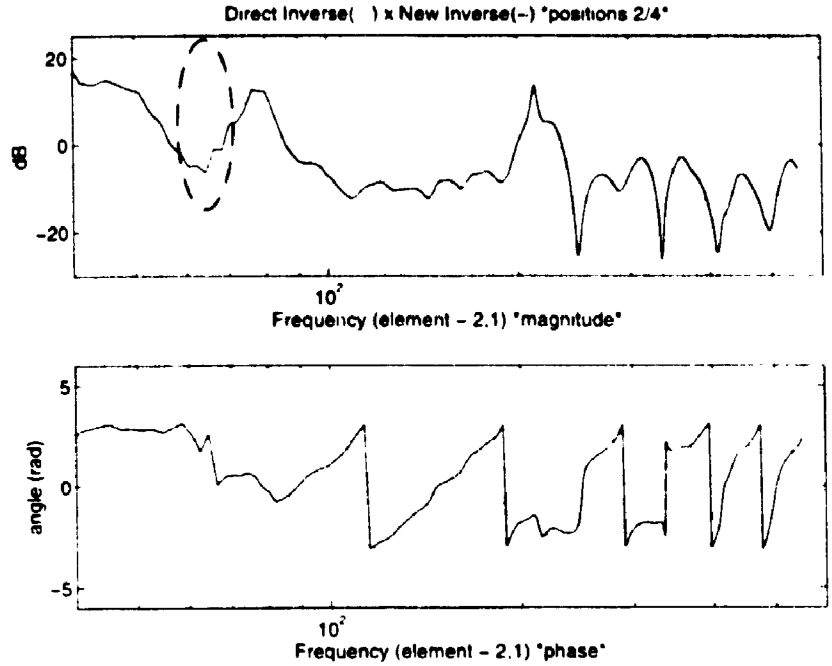

FIGURE 77. Direct inverse of H and New inverse of H - element 2,1 (equalization of positions 2 and 4)

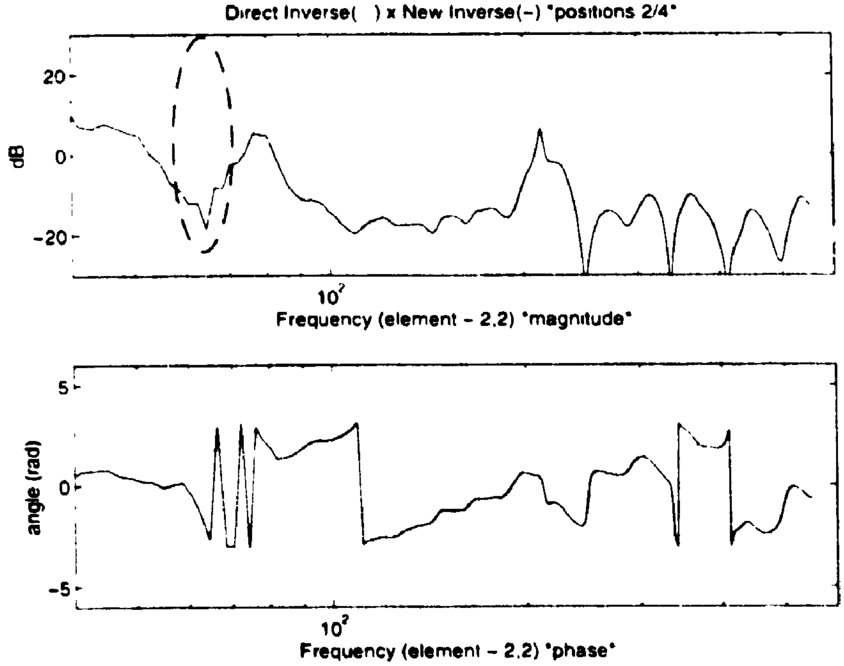

FIGURE 78. Direct inverse of H and New inverse of H - element 2,2 (equalization of positions 2 and 4)

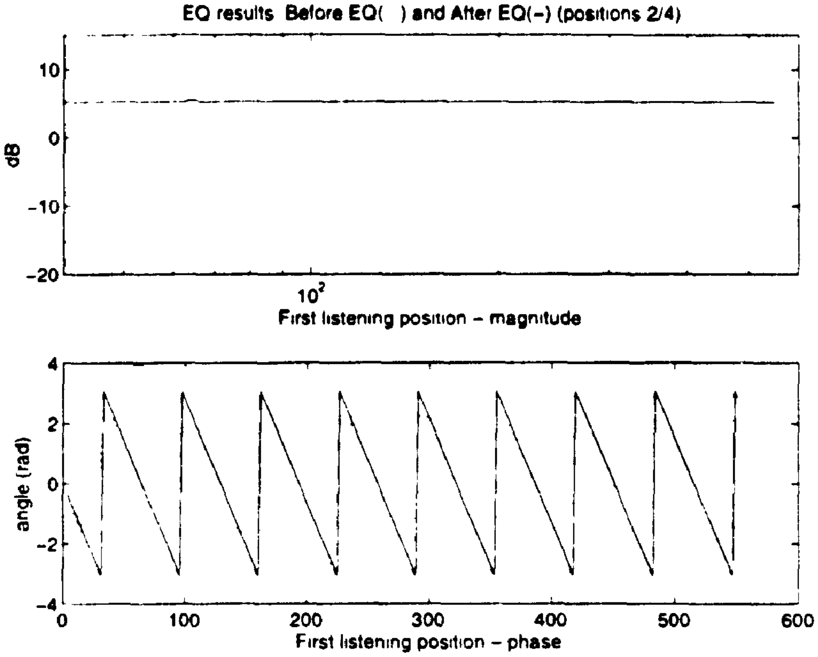

FIGURE 79. Equalization result for position 2 - using sources 2 and 4

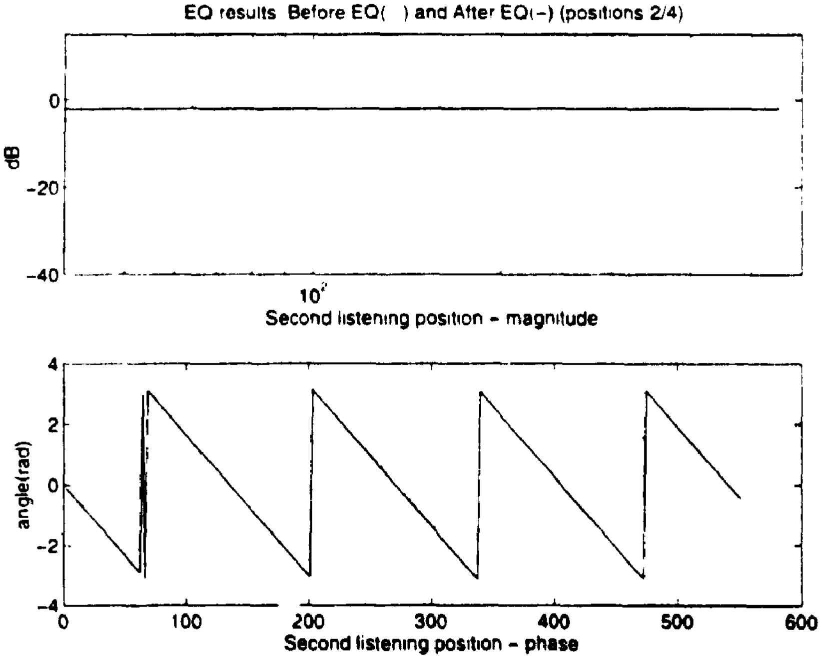

FIGURE 80. Equalization result for position 4 - using sources 2 and 4

Listening Positions 3 and 4 Using Sound Sources 3 and 4

For this case, the group of peaks between 60Hz and 80Hz is not really noticeable, and the biggest peak within the band of interest happens at around 200Hz. Big peaks for the new inverse of H happen at around 50Hz, where a smaller peak in the condition number is noticeable. Since only one problematic frequency could be used as reference for the equalization process, the one with the highest peak was chosen.

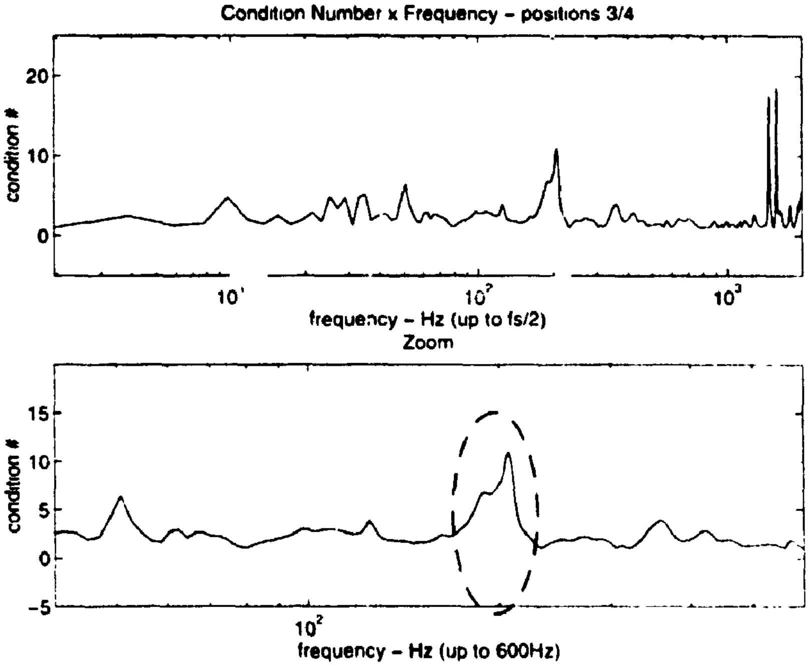

FIGURE 81. Condition Number versus frequency - positions 3 and 4

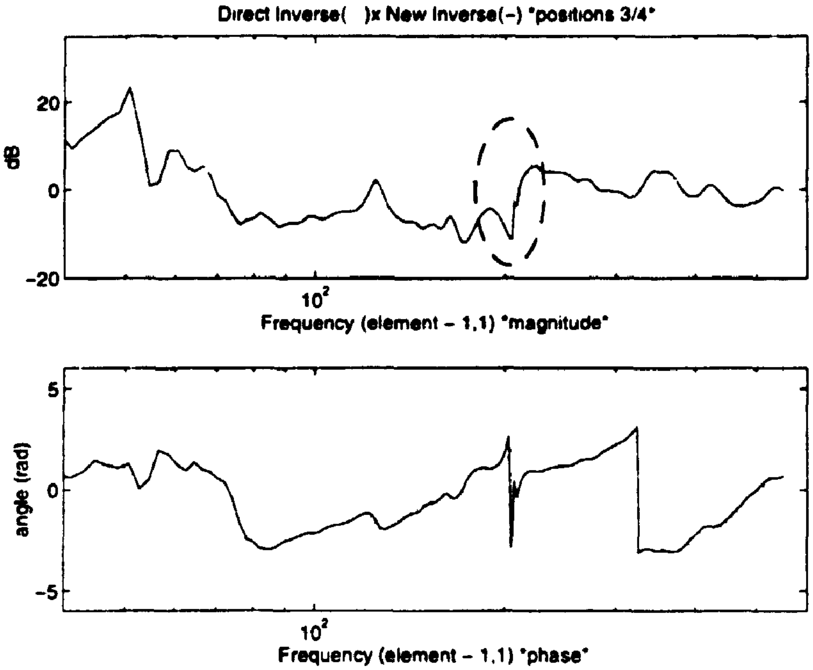

FIGURE 82. **Direct inverse of H and New inverse of H - element 1,1 (equalization of positions 3 and 4)**

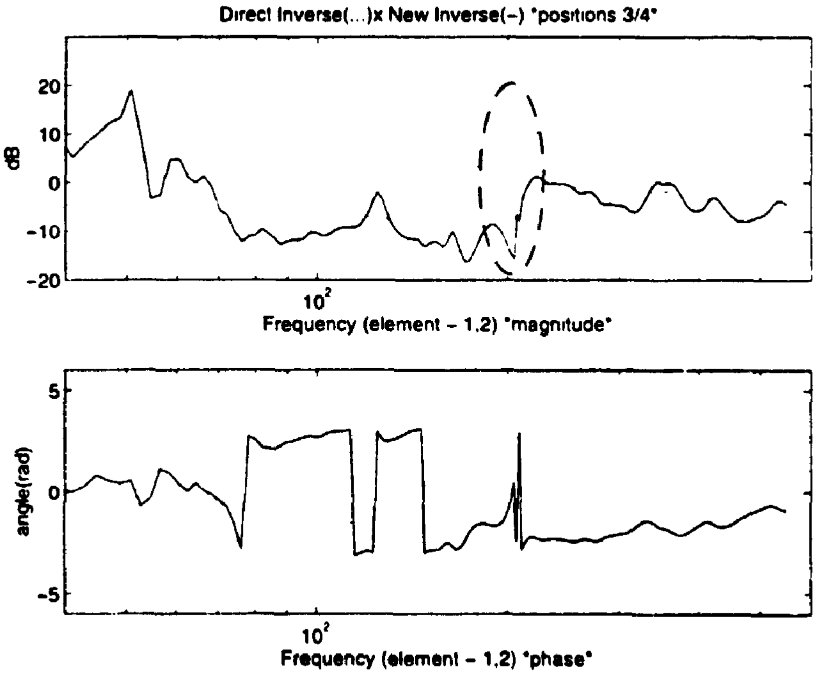

FIGURE 83. **Direct inverse of H and New inverse of H - element 1,2 (equalization of positions 3 and 4)**

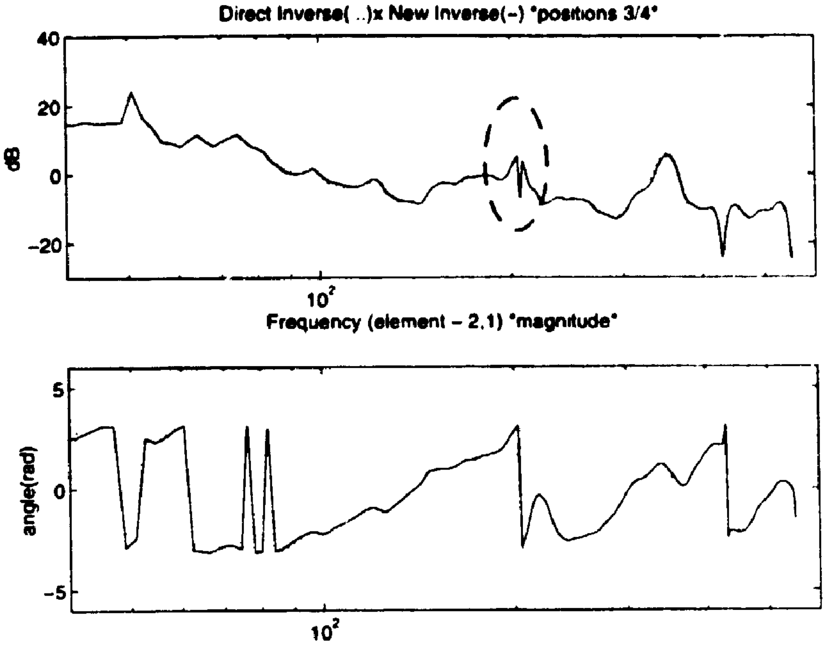

FIGURE 84. Direct inverse of H and New inverse of H - element 2,1 (equalization of positions 3 and 4)

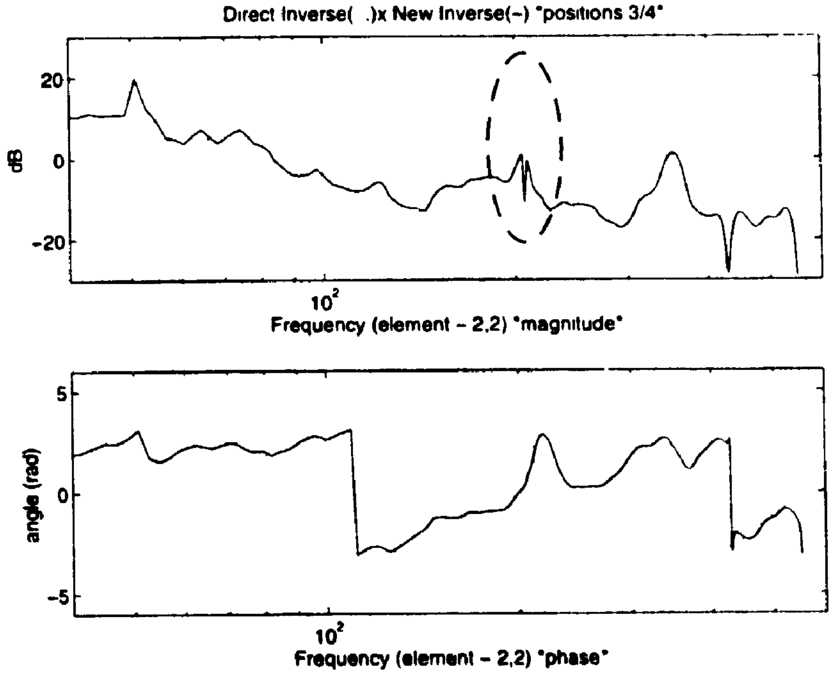

FIGURE 85. Direct inverse of H and New inverse of H - element 2,2 (equalization of positions 3 and 4)

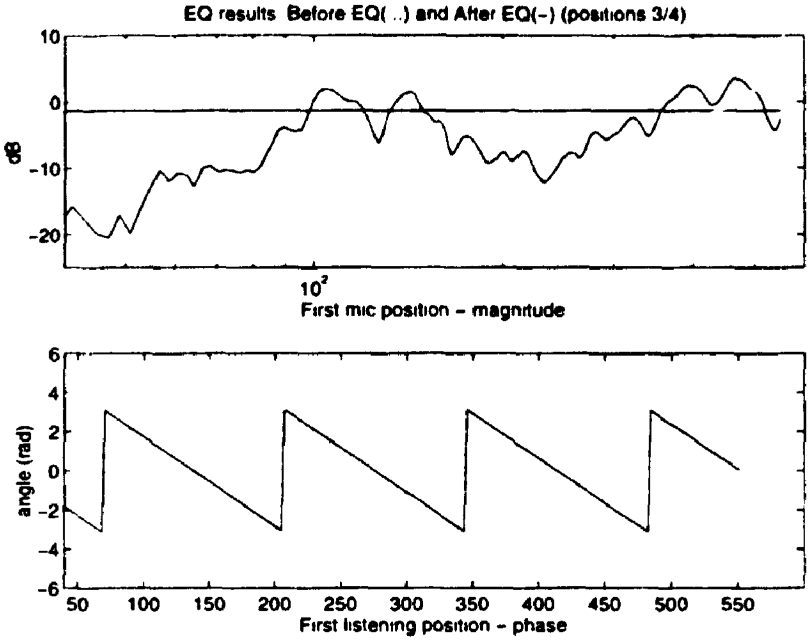

FIGURE 86. Equalization result for position 3, using sources 3 and 4

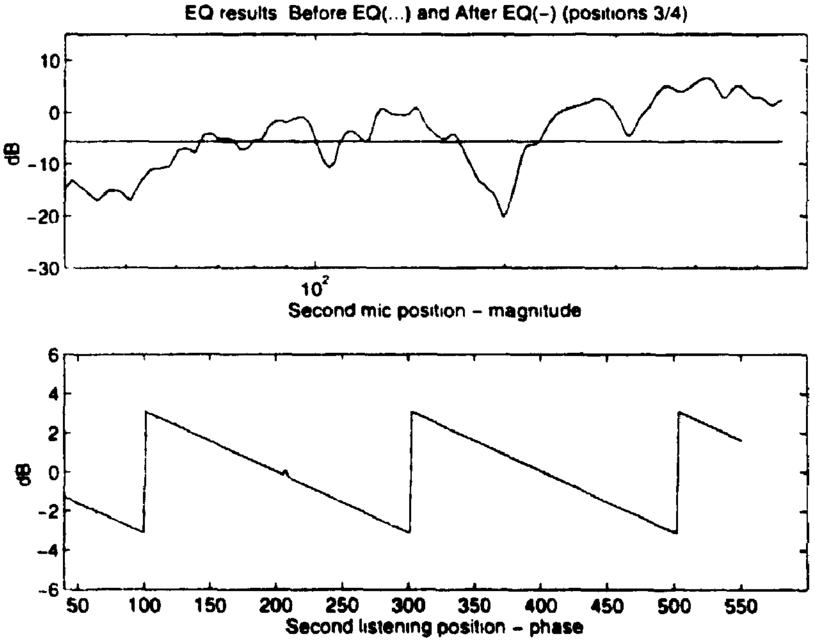

FIGURE 87. Equalization result for position 4, using sources 3 and 4

Comments on the Results

All results showed a flat overall response at each position involved, and also achieved a reduction for the peaks present in the elements of the direct inverse of transfer function matrix H at the problematic frequencies. The phase correction applied to the desired vector was shown to be effective (as tracking the original phase response at the positions), but for some configurations the corrected phase shows a greater time delay than the original one found at each of the listening positions of the venue.

The difference in magnitude (gain) at each position is quite big in some instances. That could be corrected by placing a linear amplifier before the signal is sent to the speaker, compensating the attenuation at the output of the equalizer.

Some of the configurations however presented a small feature in the overall transfer function, at the problematic frequency. Those features usually corresponded to other ones in the phase response, indicating a different phase response for one frequency only. This detail is believed to be caused by a bad choice of a desired vector. No research was done on finding an effective way of "fine tune" the desired vector to achieve the ideal flat magnitude and linear phase.

Other Configurations Tried

Other configurations were also tried, and presented good results. One example is the attempt to equalize positions 1 and 2 using sound sources 3 and 4 presented in chapter 4. Another example following the previous one is the equalization of positions 3 and 4 using sound sources 1 and 2. This second example is presented next.

In a practical approach, one can think of these configurations as loudspeakers at the back of a car being used to equalize the two front seats, or loudspeakers positioned in the front part of the car being used to equalize positions in the back seats. The impulse responses presented in Appendix A roughly imply that positions 1 and 2 are not only farther away from sources 3 and 4, but also the impulse response for those paths is much more attenuated, indicating perhaps the presence of some padding in the interior of the car where the measurements were taken. The same happens when positions 3 and 4 are to be equalized using sources 1 and 2.

Listening positions 3 and 4, Using Sound Sources 1 and 2

The following figures illustrate the attempt for sources 1 and 2 being used to equalize positions 3 and 4 (the other attempt was presented in chapter 4). Only the condition number plot and the results are presented.

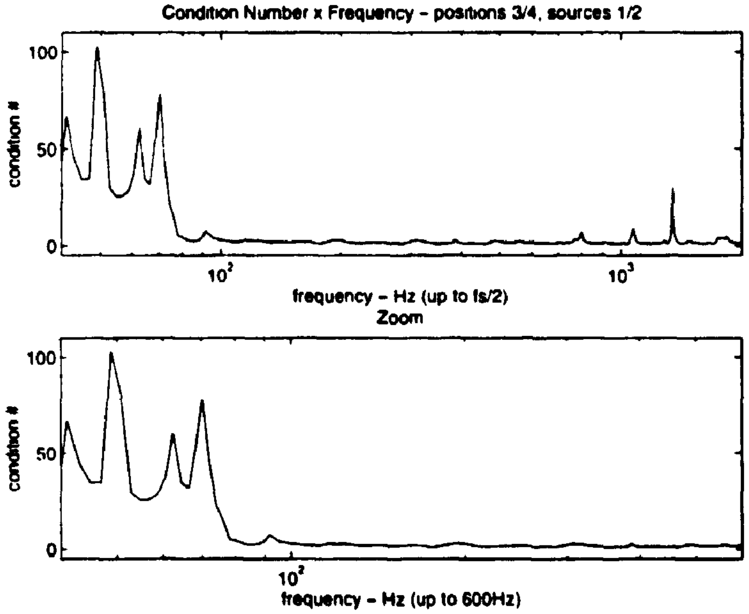

FIGURE 88. Condition Number versus Frequency, for positions 3 and 4 equalized by speakers 1 and 2

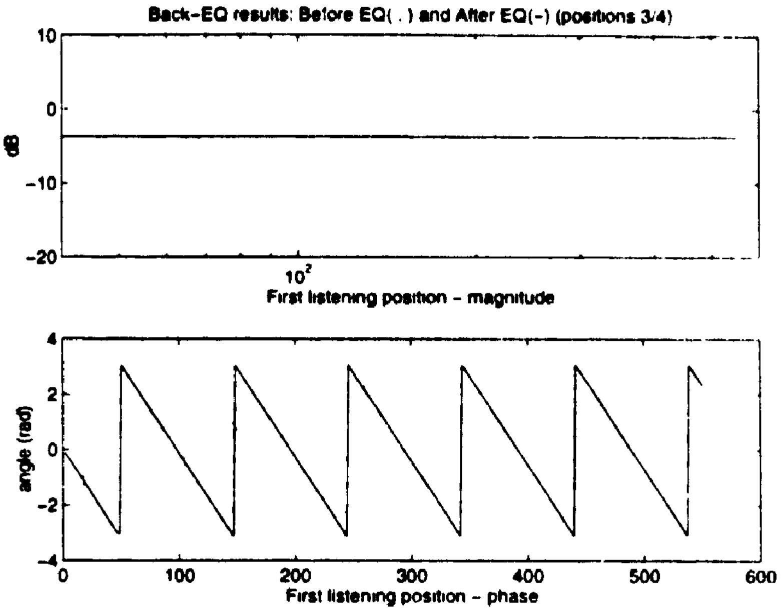

FIGURE 89. Overall response for position 3 - speakers 1 and 2 equalizing positions 3 and 4

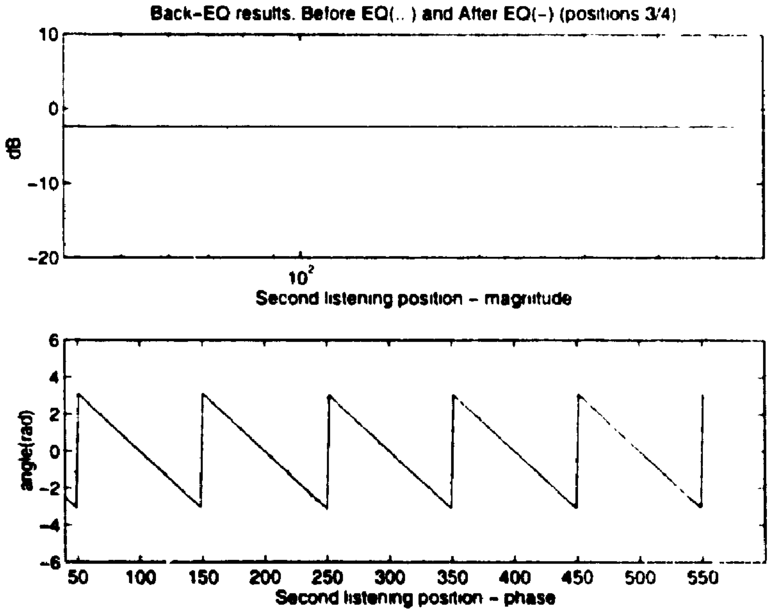

FIGURE 90. Overall response for position 4 - speakers 1 and 2 equalizing positions 3 and 4

APPENDIX C Some Routines Used

Matlab Routines

This section presents two routines written for the equalization algorithm: the correction of the eigenvector matrix, and the Gram-Schmidt orthogonalization procedure

Correcting the Eigenvector Matrix

```
%**********************************************************************
% Function to correct the eigenvector matrix according to the
% corresponding eigenvalue
%
%   Program CorrV.m          Brun FF  Jan9?
%
% This program extracts the smallest eigenvalue of the eigenvalue matrix
% and places the corresponding eigenvector on the last column of a
% "corrected" eigenvector matrix and the one associated with the biggest
% eigenvalue on the first column, prior to ortogonalization
%
%      Run this routine BEFORE orthogonalization, as the "gram" routine will
% orthogonalize the matrix having as anchor the eigenvector corresponding to
% the smallest eigenvalue
%**********************************************************************

function [Vcorr] = corrv(a,b)     % a- eigenvalue matrix (diagonal) "D" in matlab
                                  % b- eigenvector matrix (4x4)     "V" in matlab
[r dim]=size(a)

for i=1:dim                 % make a vector with the eigenvalues
  for j=1:dim
    if i==j
      tmp(i,1) = a(i,j)
    end
  end
end

tt=abs(tmp);

p=2;   %initiate a counter
pp=1   %initiate a counter for the case of two maximums
ppp=dim  %initiate another counter for the case of two minimums

for m=1:dim
  if abs(tmp(m))~=min(abs(tmp)) & abs(tmp(m))~=max(abs(tmp))
     Vc(1:dim,p) = b(1:dim,m);
     p=p+1;  %increment counter
  elseif abs(tmp(m))==max(abs(tmp))
     Vc(1:dim,pp) = b(1:dim,m);
     pp=pp+1;
  elseif abs(tmp(m))==min(abs(tmp))
     Vc(1:dim,ppp) = b(1:dim,m);
     ppp=ppp-1;
  end
end

Vcorr = Vc;
```

The Gram-Schmidt Orthogonalization Procedure

```
%*********************************************************
% FUNCTION  gram-schmidt algorithm for orthogonalization
%
%   by BrunoKF -- jan95
%
%       Usage [matrix_name matrix_name]=gram(square matrix)
%
% This function performs the orthogonalization of a given
% matrix  and extracts the inverse of that matrix to check
% if equals the transpose of the original matrix
%
%  obs  note that for matrix with complex elements  the inner product between
%       two columns makes use of the complex conjugates of one of the columns
%       In order to check the resulting matrix  such matrix should be equal
%       to its transpose  which is the "complex conjugate of the transpose"
%       of the original matrix  Finally  the norm of a vector (column) with
%       complex elements also makes use of the complex conjugate  but that
%       one is taken care of by Matlab
%
%   inner prod of complex - pg.85  Applied Linear Alg    Ben Noble
%                           pg215, Algebra Linear   Boldrini (Port)
%*******************************************************************

function[U mod] = gram(a)   % function "gram" returns matrix U and its
                            % absolute value as results

%   - --  This is what is implemented            --
%
%                         <a(1:row,k),V(1:row,j)>
%  V(1:row,k)=a(1:row,k) - ----------------------- * V(1:row,j)
%                         <V(1:row,j),V(1:row,j)>
%  - --  - ------- -- - -  --        -          - -

[row col] = size(a)     %collect the size of the input matrix
                        % "dimension of the base"

V(1:row,1)=a(1:row,1)    %first column of V equals first column of the
                         %matrix

if [illegible]
   disp('Sorry, dimension(s) too big')
end

%***********************
% for 2x2 matrices
%***********************

if col==2

   la1 = a(1:row,2)

   inpr1 = sum(a(1:row,2).*conj(V(1:row,1)))
   inpr2 = sum(V(1:row,1).*conj(V(1:row,1)))
   le2 = V(1:row,1)*(inpr1/inpr2)

   V(1:row,2)=la1-le2

   %disp('The orthonormal base is   ')

   U(1:row,1)=V(1:row,1)/norm(V(1:row,1))
   U(1:row,2)=V(1:row,2)/norm(V(1:row,2))

   U    % Finally

   %disp('compare with this')

   %test=conj(inv(U'))

end

% this routine is implemented like above for the 2x2 case  For greater
% dimensions  it is more efficiently implemented as a loop
% due to the nature of the algorithm
```

Assembly Routines

This section presents two of the filtering routines used for the real-time implementation. The first one implements the "Quadrature Mirror Filter" in an efficient way. The second implements the interpolation filters of the reconstruction stages. Notice the similarities between the two routines. This is one of the main advantages of using these filters.

QMF stage - highpass / lowpass

```
{               QMF2CH.asm    - by Bruno KF   Sept 95

        Routine that implements QMF filters using halfband filters, taking
advantage of the zeros existing as coefficients. The filtering is done for
two channels simultaneously

        Input (dline) is in i0 - left(dm) and i1 - right(dm)
        Coefficients are in b8 (pm)
        Output L and R is direct into the delay line of the next stage

        This routine was designed ONLY for 27 tap halfband filters, with
coefficients starting with zero ! It has to be modified otherwise
This could have been made a loop, but it would be worth the effort for big
filters, not for small ones
        Notice: m0, m1 and m8 need to be 2, and l8 = coeffs+1 = 10+18+taps
        PREMODIFY DOES NOT WORK WITH CIRCULAR BUFFERS  Pg 4-4 manual

        m0, i0, l0 have to do with channel L
        m1, i1, l1 have to do with channel R
}
 GLOBAL qmf2ch, qmf_init
 SEGMENT PM pm_sram
qmf2ch:
                (the modifiers m0 and m1 need to be 2)
                                dm(i0,m0)=f10 }    {bring sample in - L }
                                dm(i1,m1)=f11 }    {channel R}
                modify(i8,-7);
                f4=pm(i8,-7);     {remember i8 has been modified already!!}

                modify(i0,-7);
                modify(i1,-7);
                f0=dm(i0,-7);     {calculate centre - L }
                f1=dm(i1,-7);     {centre - R}

        {f7 and f9 are centre elements, calculated first}

                f7=f0*f4, f0=dm(i0,m0);                            {left}
                f9=f1*f4, f1=dm(i1,m1), f4=pm(i8,m8);              {right}
                f8=f0*f4, f0=dm(i0,m0);
               f10=f1*f4, f1=dm(i1,m1), f4=pm(i8,m8);
               f12=f0*f4, f0=dm(i0,m0);
               f15=f1*f4, f1=dm(i1,m1), f4=pm(i8,m8);
        f12=f0*f4,  f8=f8+f12, f0=dm(i0,m0);                       {left}
        f15=f1*f4, f10=f10+f15, f1=dm(i1,m1), f4=pm(i8,m8);        {right}
        f12=f0*f4,  f8=f8+f12, f0=dm(i0,m0);
        f15=f1*f4, f10=f10+f15, f1=dm(i1,m1), f4=pm(i8,m8);
        f12=f0*f4,  f8=f8+f12, f0=dm(i0,m0);
        f15=f1*f4, f10=f10+f15, f1=dm(i1,m1), f4=pm(i8,m8);
        f12=f0*f4,  f8=f8+f12, f0=dm(i0,m0);
        f15=f1*f4, f10=f10+f15, f1=dm(i1,m1), f4=pm(i8,m8);
        f12=f0*f4,  f8=f8+f12, f0=dm(i0,m
        f15=f1*f4, f10=f10+f15, f1=dm(i1,m1);

        f4=pm(i8,3);
        f12=f0*f4,  f8=f8+f12;       {L}
        f15=f1*f4, f10=f10+f15;      {R}
                    f0=f8+f12;       {L}
                    f1=f10+f15;      {R}

        f2=f7-f0;      {output high pass - L}
        f3=f9-f1;      {output high pass - R}
        rts(db);
        f0=f7+f0;      {low pass - L}
        f1=f9+f1;      {low pass - R}
qmf_init:
        lcntr=r0, do zero until lce;
        dm(i1,m1)=0;
zero:   dm(i0,m0)=0;
    rts;
 ENDSE
```

Interpolation Filter

```
{               FIR2CH.asm   --  by Brun  KF  Sept 9[illegible]

        Routine that implements an FIR filter using a halfband filter  taking
advantage of the zeros existing as coefficients  The filtering is done for
two channels simultaneously

    THIS IS THE INTERPOLATION FILTER  NEEDS TO GIVE A GAIN OF 2 AT THE END

        Input (dline) is in b0 - left(dm) and b1 - right(dm)
        Coefficients are in b8 (pm)
        output L and R is direct into the delay line of the next stage

        Notice  m0, m1 and m8 need to be 2  and b8=coefs+1  l0 l8=taps
        PRE MODIFY DOES NOT WORK WITH CIRCULAR BUFFERS  Pg 4 4 manual

        m0, i0, l0 have to do with channel L
        m1, i1  l1 have to do with channel R
}

 GLOBAL fir2ch  fir_init
 SEGMENT PM pm_sram
fir2ch:
                {the modifiers m0 and m1 need to be 2}
                            {dm(i0,m0)=f10 }  {bring sample in  L }
                            {dm(i1,m1)=f11 }  {channel R}
                modify(i8,7);
                f4=pm(i8,-7)    {remember i8 has been modified already!}
                modify(i0,7)
                modify(i1,7)
                f0=dm(i0,7)     {calculate centre  L }
                f1=dm(i1,7)     {centre  R}

        {f7 and f9 are centre elements  calculated first}

                f7=f0*f4  f0=dm(i0,m0)                                  {left}
                f9=f1*f4  f1=dm(i1,m1)  f4=pm(i8,m8)                    {right}
                f8=f0*f4  f0=dm(i0,m0)
               f10=f1*f4  f1=dm(i1,m1)  f4=pm(i8,m8)
               f12=f0*f4  f0=dm(i0,m0)
               f15=f1*f4  f1=dm(i1,m1)  f4=pm(i8,m8)
        f12=f0*f4   f8=f8+f12  f0=dm(i0,m0)                             {left}
        f15=f1*f4  f10=f10+f15  f1=dm(i1,m1)  f4=pm(i8,m8)              {right}
        f12=f0*f4  f8=f8+f12  f0=dm(i0,m0)
        f15=f1*f4  f10=f10+f15  f1=dm(i1,m1)  f4=pm(i8,m8)
        f12=f0*f4  f8=f8+f12  f0=dm(i0,m0)
        f15=f1*f4  f10=f10+f15, f1=dm(i1,m1)  f4=pm(i8,m8)
        f12=f0*f4  f8=f8+f12  f0=dm(i0,m0)
        f15=f1*f4  f10=f10+f15  f1=dm(i1,m1)  f4=pm(i8,m8)
        f12=f0*f4  f8=f8+f12  f0=dm(i0,m0)
        f15=f1*f4  f10=f10+f15  f1=dm(i1,m1)
        f4=pm(i8,1)
        f12=f0*f4   f8=f8+f12       {L}
        f15=f1*f4,  f10=f10+f15     {R}
                    f0=f8+f12       {L}
                    f1=f10+f15      {R}
        f8=f7+f0       {low pass - L}
        f10=f9+f1      {low pass - R}
        rts(db)
        f0=f8+f8
        f1=f10+f10          {the interpolation filter needs to give gain of 2}
fir_init:
        lcntr=[illegible], do zer until lce
        dm(i1,m1)=f0
zer     dm(i0,m0)=f0
    rts
 ENDSEG;
```